V úlohách 3.1.1. až 3.1.5. vypočítajte neurčité integrály (na intervaloch, v ktorých existujú):

3.1.1. **a)** $\int (5x^2 - 4x + 10)\,\mathrm{d}x$ **b)** $\int x \cdot (3x-4)^2\,\mathrm{d}x$

c) $\int (x^2 - 4x \cdot \sqrt[3]{x} + 10 \cdot \sqrt[4]{x^3})\,\mathrm{d}x$ **d)** $\int (x-2)(4-x)\,\mathrm{d}x$

e) $\int (x^2 - 2)^3\,\mathrm{d}x$ **f)** $\int (x^2 - \frac{5}{2}x + 6) \cdot (x^3 + 1)\,\mathrm{d}x$

g) $\int (\frac{3}{x} - \frac{7}{x^3} + \frac{6}{\sqrt[3]{x}} - \frac{18}{x\sqrt{x}})\,\mathrm{d}x$ **h)** $\int x \cdot \sqrt{x \cdot \sqrt{x}}\,\mathrm{d}x$

i) $\int (1 - 3x + x^3) \cdot \sqrt[3]{x}\,\mathrm{d}x$ **j)** $\int \sqrt{x \cdot \sqrt{x^3}}\,\mathrm{d}x$

3.1.2. **a)** $\int (e^x - e^3)\mathrm{d}x$ **b)** $\int 5^x \cdot e^x \mathrm{d}x$

c) $\int \frac{15^x - 9^x}{3^x}\mathrm{d}x$ **d)** $\int \frac{(3^x + 4^x)^2}{12^x}\mathrm{d}x$

3.1.3. **a)** $\int (\sin x - 3\cos x)\mathrm{d}x$ **b)** $\int (\operatorname{cotg}^2 x)\mathrm{d}x$

c) $\int \frac{1}{\cos^2 x \cdot \sin^2 x}\mathrm{d}x$ **d)** $\int \frac{2}{x^2 + 9}\mathrm{d}x$

3.1.4. **a)** $\int \frac{\cos x}{10 + \sin x}\mathrm{d}x$ **b)** $\int \frac{\sin x}{2 + 5 \cdot \cos x}\mathrm{d}x$

c) $\int \frac{1}{x \cdot \ln x}\mathrm{d}x$ **d)** $\int \frac{1}{\sin x \cdot \cos x}\mathrm{d}x$

3.1.5. **a)** $\int \frac{3}{2 - 5x}\mathrm{d}x$ **b)** $\int \frac{x}{9 + 4x^2}\mathrm{d}x$

V úlohách 3.2.1. až 3.2.3. vypočítajte neurčité integrály:

3.2.1. **a)** $\int \frac{\ln^4 x}{x}\mathrm{d}x$ **b)** $\int \frac{1}{x^2} \cdot \cos\frac{1}{x}\mathrm{d}x$

c) $\int x^2 \cdot e^{x^3}\mathrm{d}x$ **d)** $\int x \cdot (3x^2 - 4)^5\mathrm{d}x$

e) $\int \frac{x}{(x^2 - 4)^3}\,\mathrm{d}x$ **f)** $\int x^2 \cdot \sqrt[3]{6 - x^3}\,\mathrm{d}x$

g) $\int \frac{1}{x^3} \cdot \sin\frac{1}{x^2}\mathrm{d}x$ **h)** $\int \frac{3x^3}{\sqrt[3]{x^4 + 4}}\mathrm{d}x$

3.2.2. **a)** $\int \sqrt{5 + 2x}\,\mathrm{d}x$ **b)** $\int \frac{5}{\sqrt[3]{1 - 6x}}\mathrm{d}x$

c) $\int \sin\left(\frac{3x - 5}{2}\right)\mathrm{d}x$ **d)** $\int \frac{1}{\sin^2\left(\frac{x-2}{3}\right)}\,\mathrm{d}x$

e) $\int \operatorname{cotg}(5x+9)\,\mathrm{d}x$

f) $\int \frac{3}{\sqrt{(5-2x)^3}}\,\mathrm{d}x$

g) $\int (3-2x)^3\,\mathrm{d}x$

h) $\int \frac{1}{(5+3x)^3}\,\mathrm{d}x$

3.2.3. a) $\int \frac{2^x}{1+4^x}\,\mathrm{d}x$

b) $\int \frac{x^2}{\sqrt{1-x^6}}\,\mathrm{d}x$

c) $\int \frac{e^{\frac{1}{x}}}{x^2}\,\mathrm{d}x$

d) $\int e^{\cos^2 x}\cdot \sin 2x\,\mathrm{d}x$

e) $\int \frac{\sqrt{1+\ln x}}{x}\,\mathrm{d}x$

f) $\int \frac{1}{x\cdot\sqrt[3]{\ln 3x}}\,\mathrm{d}x$

g) $\int \frac{\operatorname{tg}\sqrt{x}}{\sqrt{x}}\,\mathrm{d}x$

h) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,\mathrm{d}x$

V úlohách 3.3.1. až 3.3.2. vypočítajte neurčité integrály.

3.3.1. a) $\int x\cdot\sin x\,\mathrm{d}x$

b) $\int x\cdot e^{2x}\,\mathrm{d}x$

c) $\int (2x-5)\cdot\sin 3x\,\mathrm{d}x$

d) $\int (5x+2)\cdot 2^x\,\mathrm{d}x$

e) $\int \frac{x}{\sin^2 x}\,\mathrm{d}x$

f) $\int (x^2+6x-7)\cdot\cos x\,\mathrm{d}x$

g) $\int (4x-x^2)\cdot 5^x\,\mathrm{d}x$

h) $\int (x^2+2x-3)\cdot e^{-x}\,\mathrm{d}x$

i) $\int (3x+5)\cdot\cos\frac{x}{3}\,\mathrm{d}x$

j) $\int (2x-7)\cdot\operatorname{tg}^2 x\,\mathrm{d}x$

3.3.2. a) $\int x^3\cdot\ln x\,\mathrm{d}x$

b) $\int \operatorname{arccotg} x\,\mathrm{d}x$

c) $\int \arccos x\,\mathrm{d}x$

d) $\int \operatorname{arccotg} 2x\,\mathrm{d}x$

e) $\int x\cdot\ln^2 x\,\mathrm{d}x$

f) $\int \frac{\ln x}{x^3}\,\mathrm{d}x$

g) $\int \arcsin(2x+1)\,\mathrm{d}x$

h) $\int x^2\cdot\operatorname{arctg} x\,\mathrm{d}x$

3.3.3. Určte funkciu celkových nákladov *C*(*x*) spĺňajúcu uvedenú podmienku, ak je daná funkcia marginálnych nákladov *MC*(*x*).

a) $MC(x) = 100 - 2x$ — $FC = 1\,000$ p. j.

b) $MC(x) = 5 + 2x - 0{,}6x^2$ — $C(5) = 35$ p. j.

c) $MC(x) = \frac{1}{\sqrt{4x+256}}$ — $C(36) = 6$ p. j.

d) $MC(x) = 20\cdot e^{\frac{x}{2}}$ — $FC = 120$ p. j.

e) $MC(x) = (2x+3)\cdot e^{2x}$ — $FC = 5$ p. j.

f) $MC(x) = 12+\frac{300}{(x+1)^2}$ — $C(11) = 507$ p. j.

9.1

j) $\circledast = \dfrac{\frac{2-2\ln x}{x}\cdot x^2 - 2x(2\ln x - \ln^2 x)}{x^4} = \dfrac{x\cdot[2-2\ln x - 4\ln x + 2\ln^2 x]}{x^4} = \dfrac{2\ln^2 x - 6\ln x + 2}{x^3}$

$y''(1) = \dfrac{2\cdot \ln^2 1 - 6\cdot \ln 1 + 2}{1} = \dfrac{2}{1} = 2 > 0 \Rightarrow$ v $x_0 = 1$ je lok. minimum

$y''(e^2) = \dfrac{2\ln^2 e^2 - 6\ln e^2 + 2}{(e^2)^3} = \dfrac{2\cdot 4 - 6\cdot 2 + 2}{e^6} = \dfrac{-2}{e^6} < 0 \Rightarrow$ v $x_0 = e^2$ je lok. maximum

3.1.1

a) $\int (5x^2 - 4x + 10)\,dx = 5\cdot\frac{x^3}{3} - 4\cdot\frac{x^2}{2} + 10x = \frac{5}{3}x^3 - 2x^2 + 10x + c;\ c\in R$

b) $\int x\cdot(3x-4)^2\,dx = \int x\cdot(9x^2 - 24x + 16)\,dx = \int (9x^3 - 24x^2 + 16x)\,dx = 9\cdot\frac{x^4}{4} - 24\cdot\frac{x^3}{3} + 16\cdot\frac{x^2}{2} = \frac{9}{4}x^4 - 8x^3 + 8x^2 + c;\ c\in R$

c) $\int (x^2 - 4x\sqrt[3]{x} + 10\sqrt[4]{x^3})\,dx = \int \left(x^2 - 4x\cdot x^{\frac{1}{3}} + 10\cdot x^{\frac{3}{4}}\right)dx = \int \left(x^2 - 4x^{\frac{4}{3}} + 10x^{\frac{3}{4}}\right)dx = \frac{x^3}{3} - 4\cdot\frac{x^{\frac{7}{3}}}{\frac{7}{3}} + 10\cdot\frac{x^{\frac{7}{4}}}{\frac{7}{4}} + c =$

$= \frac{x^3}{3} - 4\cdot\frac{3}{7}\cdot x^{\frac{7}{3}} + 10\cdot\frac{4}{7}\cdot x^{\frac{7}{4}} + c = \frac{1}{3}x^3 - \frac{12}{7}\cdot\sqrt[3]{x^7} + \frac{40}{7}\cdot\sqrt[4]{x^7} + c;\ c\in R$

d) $\int (x-2)(4-x)\,dx = \int (4x - x^2 - 8 + 2x)\,dx = \int (-x^2 + 6x - 8)\,dx = -\frac{x^3}{3} + 6\cdot\frac{x^2}{2} - 8x = -\frac{1}{3}x^3 + 3x^2 - 8x + c;\ c\in R$

e) $\int (x^2-2)^3\,dx = \int (x^2-2)^2\cdot(x^2-2)\,dx = \int (x^4 - 4x^2 + 4)(x^2-2)\,dx = \int (x^6 - 2x^4 - 4x^4 + 8x^2 + 4x^2 - 8)\,dx = \int (x^6 - 6x^4 + 12x^2 - 8)\,dx =$

$= \frac{x^7}{7} - 6\cdot\frac{x^5}{5} + 12\cdot\frac{x^3}{3} - 8x + c = \frac{1}{7}x^7 - \frac{6}{5}x^5 + 4x^3 - 8x + c;\ c\in R$

f) $\int \left(x^2-\frac{5}{2}x+6\right)(x^3+1)\,dx = \int\left(x^5+x^2-\frac{5}{2}x^4-\frac{5}{2}x+6x^3+6\right)dx = \int\left(x^5-\frac{5}{2}x^4+6x^3+x^2-\frac{5}{2}x+6\right)dx =$

$= \frac{x^6}{6}-\frac{5}{2}\cdot\frac{x^5}{5}+6\cdot\frac{x^4}{4}+\frac{x^3}{3}-\frac{5}{2}\cdot\frac{x^2}{2}+6x+c = \frac{1}{6}x^6-\frac{1}{2}x^5+\frac{3}{2}x^4+\frac{1}{3}x^3-\frac{5}{4}x^2+6x+c;\ c\in R$

g) $\int\left(\frac{3}{x}-\frac{7}{x^3}+\frac{6}{\sqrt[3]{x}}-\frac{18}{x\sqrt{x}}\right)dx = \int\left(3\cdot\frac{1}{x}-7\cdot\frac{1}{x^3}+6\cdot\frac{1}{x^{\frac{1}{3}}}-18\cdot\frac{1}{x\cdot x^{\frac{1}{2}}}\right)dx = \int\left(3x^{-1}-7x^{-3}+6x^{-\frac{1}{3}}-18x^{-\frac{3}{2}}\right)dx =$

$= 3\ln x - 7\cdot\frac{x^{-2}}{-2}+6\cdot\frac{x^{\frac{2}{3}}}{\frac{2}{3}}-18\cdot\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+c =$

$= 3\ln x+\frac{7}{2}\cdot\frac{1}{x^2}+6\cdot\frac{3}{2}\cdot\sqrt[3]{x^2}+18\cdot 2\cdot\frac{1}{\sqrt{x}}+c = 3\ln x+\frac{7}{2x^2}+9\sqrt[3]{x^2}+\frac{36}{\sqrt{x}}+c$

h) $\int\left(x\cdot\sqrt{x\cdot\sqrt{x}}\right)dx = \int x\cdot\left(x\cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}}dx = \int x\cdot x^{\frac{1}{2}}\cdot x^{\frac{1}{4}}dx = \int x^{1+\frac{1}{2}+\frac{1}{4}}dx = \int x^{\frac{4+2+1}{4}}dx = \int x^{\frac{7}{4}}dx = \frac{x^{\frac{11}{4}}}{\frac{11}{4}} = \frac{4}{11}\cdot\sqrt[4]{x^{11}}+c$

i) $\int(1-3x+x^3)(\sqrt[3]{x})\,dx = \int(1-3x+x^3)\cdot x^{\frac{1}{3}}dx = \int\left(x^{\frac{1}{3}}-3\cdot x^{\frac{4}{3}}+x^{\frac{10}{3}}\right)dx = \frac{x^{\frac{4}{3}}}{\frac{4}{3}}-3\cdot\frac{x^{\frac{7}{3}}}{\frac{7}{3}}+\frac{x^{\frac{13}{3}}}{\frac{13}{3}}+c =$

$= \frac{3}{4}\cdot\sqrt[3]{x^4}-3\cdot\frac{3}{7}\cdot\sqrt[3]{x^7}+\frac{3}{13}\cdot\sqrt[3]{x^{13}}+c = \frac{3}{4}\cdot\sqrt[3]{x^4}-\frac{9}{7}\cdot\sqrt[3]{x^7}+\frac{3}{13}\cdot\sqrt[3]{x^{13}}+c;\ c\in R$

j) $\int\sqrt{x\cdot\sqrt{x^3}}\,dx = \int\left(x\cdot x^{\frac{3}{2}}\right)^{\frac{1}{2}}dx = \int\left(x^{\frac{5}{2}}\right)^{\frac{1}{2}}dx = \int x^{\frac{5}{4}}dx = \frac{x^{\frac{9}{4}}}{\frac{9}{4}}+c = \frac{4}{9}\cdot\sqrt[4]{x^9}+c;\ c\in R$

3.1.2

a) $\int (e^x - e^3)\,dx = e^x - e^3 \cdot x + c;\ c \in R$

b) $\int (5^x \cdot e^x)\,dx = \int (5e)^x\,dx = \frac{(5e)^x}{\ln(5e)} + c;\ c \in R$

c) $\int \frac{15^x - 9^x}{3^x}\,dx = \int \left(\frac{15^x}{3^x} - \frac{9^x}{3^x}\right)dx = \int \left[\left(\frac{15}{3}\right)^x - \left(\frac{9}{3}\right)^x\right]dx = \int (5^x - 3^x)\,dx = \frac{5^x}{\ln 5} - \frac{3^x}{\ln 3} + c;\ c \in R$

d) $\int \frac{(3^x + 4^x)^2}{12^x}\,dx = \int \frac{(3^x)^2 + 2\cdot 3^x \cdot 4^x + (4^x)^2}{12^x}\,dx = \int \frac{(3^2)^x + 2\cdot(3\cdot 4)^x + (4^2)^x}{12^x}\,dx = \int \left[\left(\frac{9}{12}\right)^x + 2 + \left(\frac{16}{12}\right)^x\right]dx = \int \left[\left(\frac{3}{4}\right)^x + 2 + \left(\frac{4}{3}\right)^x\right]dx =$

$= \frac{\left(\frac{3}{4}\right)^x}{\ln \frac{3}{4}} + 2x + \frac{\left(\frac{4}{3}\right)^x}{\ln \frac{4}{3}} + c;\ c \in R$

3.1.3

$\boxed{\sin^2 x + \cos^2 x = 1}$

a) $\int (\sin x - 3\cos x)\,dx = \int \sin x\,dx - 3\cdot \int \cos x\,dx = -\cos x - 3\cdot \sin x + c;\ c \in R$

b) $\int (\operatorname{cotg}^2 x)\,dx = \int \left(\frac{\cos x}{\sin x}\right)^2 dx = \int \frac{\cos^2 x}{\sin^2 x}\,dx = \int \frac{1 - \sin^2 x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x}\,dx - \int 1\,dx = -\operatorname{cotg} x - x + c;\ c \in R$

c) $\int \frac{1}{\cos^2 x \cdot \sin^2 x}\,dx = \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \cdot \sin^2 x}\,dx = \int \frac{\sin^2 x}{\cos^2 x \sin^2 x}\,dx + \int \frac{\cos^2 x}{\cos^2 x \sin^2 x}\,dx = \int \frac{1}{\cos^2 x}\,dx + \int \frac{1}{\sin^2 x}\,dx = \operatorname{tg} x - \operatorname{cotg} x + c$

d) $\int \frac{2}{x^2 + 9}\,dx = 2\cdot \int \frac{1}{9\cdot\left(\frac{x^2}{9} + 1\right)}\,dx = \frac{2}{9}\int \frac{1}{\left(\frac{x}{3}\right)^2 + 1}\,dx = \left| \begin{array}{l} \text{subst.:} \\ \frac{x}{3} = t \\ \frac{1}{3}dx = dt \end{array} \right| = \frac{2}{9}\cdot \int \frac{1}{t^2 + 1}\cdot 3\,dt = \frac{2}{3}\operatorname{arctg} t = \frac{2}{3}\operatorname{arctg} \frac{x}{3} + c$

3.1.4

a) $\int \frac{\cos x}{10+\sin x}\,dx = \left| \begin{array}{l} \text{subst:} \\ 10+\sin x = t \\ (10+\sin x)'\,dx = dt \\ \cos x\,dx = dt \end{array} \right| = \int \frac{1}{t}\,dt = \ln|t| + c = \underline{\underline{\ln|10+\sin x| + c;\ c\in R}}$

b) $\int \frac{\sin x}{2+5\cos x}\,dx = -\frac{1}{5}\int \frac{-5\sin x}{2+5\cos x}\,dx = \underline{\underline{-\frac{1}{5}\ln|2+5\cos x| + c;\ c\in R}} \Rightarrow$ lebo: $\boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c}$

čitateľ je deriváciou menovateľa

vynásobíme aj vydelíme číslom (-5), aby sme v čitateli dostali deriváciu menovateľa

c) $\int \frac{1}{x\ln x}\,dx = \left| \begin{array}{l} \ln x = t \\ \frac{1}{x}\,dx = dt \end{array} \right| = \int \frac{1}{t}\,dt = \ln|t| + c = \underline{\underline{\ln|\ln x| + c;\ c\in R}}$

d) $\int \frac{1}{\sin x\cdot\cos x}\,dx = \int \frac{\sin^2 x+\cos^2 x}{\sin x\cdot\cos x}\,dx = \int\left(\frac{\sin^2 x}{\sin x\cos x} + \frac{\cos^2 x}{\sin x\cos x}\right)dx = \int\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)dx = \underline{\underline{-\ln|\cos x| + \ln|\sin x| + c}}$

3.1.5

a) $\int \frac{3}{2-5x}\,dx = -\frac{3}{5}\int \frac{-5}{2-5x}\,dx = -\frac{3}{5}\ln|2-5x| + c;\ c\in\mathbb{R}$

b) $\int \frac{x}{9+4x^2}\,dx = \frac{1}{8}\int \frac{8x}{9+4x^2}\,dx = \frac{1}{8}\ln|9+4x^2| + c;\ c\in\mathbb{R}$

3.2.1

a) $\int \frac{\ln^4 x}{x}\,dx = \left| \begin{matrix} \ln x = t \\ \frac{1}{x}dx = dt \end{matrix} \right| = \int t^4\,dt = \frac{t^5}{5} + c = \frac{\ln^5 x}{5} + c;\ c\in\mathbb{R}$

b) $\int \frac{1}{x^2}\cdot\cos\frac{1}{x}\,dx = \left| \begin{matrix} \frac{1}{x} = t \\ -\frac{1}{x^2}dx = dt \end{matrix} \right| = -\int \cos t\,dt = -\sin t + c = -\sin\frac{1}{x} + c;\ c\in\mathbb{R}$

c) $\int x^2\cdot e^{x^3}\,dx = \left| \begin{matrix} x^3 = t \\ 3x^2dx = dt \end{matrix} \right| = \frac{1}{3}\int 3x^2 e^{x^3}\,dx = \frac{1}{3}\int e^t\,dt = \frac{1}{3}e^t + c = \frac{1}{3}e^{x^3} + c;\ c\in\mathbb{R}$

d) $\int x\cdot(3x^2-4)^5\,dx = \left| \begin{matrix} 3x^2-4 = t \\ 6x\,dx = dt \end{matrix} \right| = \frac{1}{6}\int 6x\cdot(3x^2-4)^5\,dx = \frac{1}{6}\int t^5\,dt = \frac{1}{6}\cdot\frac{t^6}{6} + c = \frac{(3x^2-4)^6}{36} + c;\ c\in\mathbb{R}$

e) $\int \frac{x}{(x^2-4)^3}\,dx = \left| \begin{matrix} x^2-4 = t \\ 2x\,dx = dt \end{matrix} \right| = \frac{1}{2}\int \frac{1}{t^3}\,dt = \frac{1}{2}\int t^{-3}\,dt = \frac{1}{2}\cdot\frac{t^{-2}}{-2} + c = \frac{-1}{4\cdot(x^2-4)^2} + c;\ c\in\mathbb{R}$

3.2.1

f) $\int x^2 \cdot \sqrt[3]{6-x^3}\,dx = \left| \begin{matrix} 6-x^3=t \\ -3x^2dx=dt \end{matrix} \right| = -\frac{1}{3}\int \sqrt[3]{t}\,dt = -\frac{1}{3}\int t^{\frac{1}{3}}dt = -\frac{1}{3}\cdot \frac{t^{\frac{4}{3}}}{\frac{4}{3}} + c = -\frac{1}{4}\cdot\sqrt[3]{(6-x^3)^4} + c$

g) $\int \frac{1}{x^3}\cdot \sin\frac{1}{x^2}\,dx = \int x^{-3}\cdot \sin x^{-2}\,dx = \left| \begin{matrix} x^{-2}=t \\ -2x^{-3}dx=dt \end{matrix} \right| = -\frac{1}{2}\int \sin t\,dt = +\frac{1}{2}\cos\frac{1}{x^2} + c;\ c\in R$

h) $\int \frac{3x^3}{\sqrt[3]{x^4+4}}\,dx = \left| \begin{matrix} x^4+4=t \\ 4x^3dx=dt \end{matrix} \right| = \frac{3}{4}\int \frac{4x^3}{(x^4+4)^{\frac{1}{3}}}\,dx = \frac{3}{4}\cdot\int \frac{1}{t^{\frac{1}{3}}}\,dt = \frac{3}{4}\cdot\int t^{-\frac{1}{3}}dt = \frac{3}{4}\cdot\frac{t^{\frac{2}{3}}}{\frac{2}{3}} = \frac{9}{8}\sqrt[3]{(x^4+4)^2}+c$

3.2.2

a) $\int \sqrt{5+2x}\,dx = \left| \begin{matrix} 5+2x=t \\ 2dx=dt \end{matrix} \right| = \frac{1}{2}\int t^{\frac{1}{2}}dt = \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{1}{3}\cdot\sqrt{(5+2x)^3} + c;\ c\in R$

b) $\int \frac{5}{\sqrt[3]{1-6x}}\,dx = \left| \begin{matrix} 1-6x=t \\ -6dx=dt \end{matrix} \right| = -\frac{5}{6}\int \frac{1}{t^{\frac{1}{3}}}\,dt = -\frac{5}{6}\cdot\frac{t^{\frac{2}{3}}}{\frac{2}{3}} + c = -\frac{5}{4}\sqrt[3]{(1-6x)^2} + c;\ c\in R$

c) $\int \sin\left(\frac{3x-5}{2}\right)dx = \left| \begin{matrix} \frac{3x-5}{2}=t \\ \frac{1}{2}\cdot 3\,dx=dt \end{matrix} \right| = \frac{2}{3}\int \sin t\,dt = -\frac{2}{3}\cos\left(\frac{3x-5}{2}\right) + c;\ c\in R$

d) $\int \frac{1}{\sin^2\left(\frac{x-2}{3}\right)}\,dx = \left| \begin{matrix} \frac{x-2}{3}=t \\ \frac{1}{3}dx=dt \end{matrix} \right| = 3\int \frac{1}{\sin^2 t}\,dt = -3\cot g\left(\frac{x-2}{3}\right) + c;\ c\in R$

e) $\int \cot g(5x+9)\,dx = \left| \begin{matrix} 5x+9=t \\ 5dx=dt \end{matrix} \right| = \frac{1}{5}\int \frac{\cos t}{\sin t}\,dt = \frac{1}{5}\ln|\sin(5x+9)| + c;\ c\in R$

100

3.2.2

f) $\int \frac{3}{\sqrt{(5-2x)^3}}\,dx = \left| \begin{array}{l} 5-2x=t \\ -2dx=dt \end{array} \right| = -\frac{1}{2}\cdot 3\cdot \int \frac{1}{t^{\frac{3}{2}}}\,dt = -\frac{3}{2}\cdot \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c = -\frac{3}{5}\cdot\sqrt{(5-2x)^5} + c;\ c\in R$

g) $\int (3-2x)^3\,dx = \left| \begin{array}{l} 3-2x=t \\ -2dx=dt \end{array} \right| = -\frac{1}{2}\int t^3\,dt = -\frac{1}{2}\,\frac{t^4}{4} + c = -\frac{1}{8}\cdot(3-2x)^4 + c;\ c\in R$

h) $\int \frac{1}{(5+3x)^3}\,dx = \left| \begin{array}{l} 5+3x=t \\ 3dx=dt \end{array} \right| = \frac{1}{3}\int \frac{1}{t^3}\,dt = \frac{1}{3}\cdot\frac{t^{-2}}{-2} + c = -\frac{1}{6}\cdot\frac{1}{(5+3x)^2} + c;\ c\in R$

3.2.3

a) $\int \frac{2^x}{1+4^x}\,dx = \left| \begin{array}{l} 2^x=t \\ 2^x\cdot\ln 2\,dx=dt \end{array} \right| = \frac{1}{\ln 2}\cdot\int \frac{2^x\cdot\ln 2}{1+(2^x)^2}\,dx = \frac{1}{\ln 2}\cdot\int \frac{1}{1+t^2}\,dt = \frac{1}{\ln 2}\cdot \operatorname{arctg} 2^x + c;\ c\in R$

b) $\int \frac{x^2}{\sqrt{1-x^6}}\,dx = \left| \begin{array}{l} x^3=t \\ 3x^2dx=dt \end{array} \right| = \frac{1}{3}\int \frac{3x^2}{\sqrt{1-(x^3)^2}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1-t^2}}\,dt = \frac{1}{3}\arcsin x^3 + c;\ c\in R$

c) $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \left| \begin{array}{l} x^{-1}=t \\ -x^{-2}dx=dt \end{array} \right| = -\int \frac{e^t}{1}\,dt = -e^{\frac{1}{x}} + c;\ c\in R$

d) $\int e^{\cos^2 x}\cdot\sin 2x\,dx = \left| \begin{array}{l} \cos^2 x=t \\ 2\cos x\cdot(-\sin x)\,dx=dt \end{array} \right| = -\int e^t\,dt = -e^{\cos^2 x} + c;\ c\in R$

platí: $\boxed{\sin 2x = 2\cdot\sin x\cdot\cos x}$

3.2.3

e) $\int \frac{\sqrt{1+\ln x}}{x}\,dx = \left| \begin{matrix} 1+\ln x = t \\ \frac{1}{x}dx = dt \end{matrix} \right| = \int t^{\frac{1}{2}}\,dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}\cdot\sqrt{(1+\ln x)^3} + c;\ c\in\mathbb{R}$

f) $\int \frac{1}{x\cdot\sqrt[3]{\ln 3x}}\,dx = \left| \begin{matrix} \ln 3x = t \\ \frac{1}{3x}\cdot 3dx = dt \end{matrix} \right| = \int \frac{1}{\sqrt[3]{t}}\,dt = \int t^{-\frac{1}{3}}\,dt = \frac{t^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{3}{2}\cdot\sqrt[3]{(\ln 3x)^2} + c;\ c\in\mathbb{R}$

g) $\int \frac{\operatorname{tg}\sqrt{x}}{\sqrt{x}}\,dx = \left| \begin{matrix} x^{\frac{1}{2}} = t \\ \frac{1}{2}x^{-\frac{1}{2}}dx = dt \end{matrix} \right| = 2\int \operatorname{tg} t\,dt = 2\cdot\int \frac{\sin t}{\cos t}\,dt = -2\ln|\cos\sqrt{x}| + c;\ c\in\mathbb{R}$

h) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx = \left| \begin{matrix} \sin x = t \\ \cos x\,dx = dt \end{matrix} \right| = \int \frac{1}{t^{\frac{2}{3}}}\,dt = \frac{t^{\frac{1}{3}}}{\frac{1}{3}} + c = 3\sqrt[3]{\sin x} + c;\ c\in\mathbb{R}$

3.3.1

a) $\int x\cdot\sin x\,dx = \left| \begin{matrix} f' = \sin x & g = x \\ f = -\cos x & g' = 1 \end{matrix} \right| = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + c;\ c\in\mathbb{R}$

Platí: $(f\cdot g)' = f'g + fg' \;/ -fg'$

$f'\cdot g = (f\cdot g)' - f\cdot g' \;/ \int$

$\int f'\cdot g\,dx = \int (f\cdot g)'\,dx - \int f\cdot g'\,dx$

$\boxed{\int f'(x)\cdot g(x)\,dx = f(x)\cdot g(x) - \int f(x)\cdot g'(x)\,dx} \rightarrow$ Per Partes

3.3.1

b) $\int x\cdot e^{2x}\,dx = \left| \begin{array}{ll} f' = e^{2x} & g = x \\ f = \frac{e^{2x}}{2} & g' = 1 \end{array}\right| = x\cdot\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx = \frac{x}{2}\cdot e^{2x} - \frac{1}{2}\int e^{2x}\,dx = \frac{x}{2}e^{2x} - \frac{1}{2}\cdot\frac{e^{2x}}{2} + c =$

$= \left(\frac{x}{2} - \frac{1}{4}\right)\cdot e^{2x} + c;\ c\in\mathbb{R}$

c) $\int (2x-5)\cdot\sin 3x\,dx = \left| \begin{array}{ll} f' = \sin 3x & g = 2x-5 \\ f = \frac{-\cos 3x}{3} & g' = 2 \end{array}\right| = -\frac{\cos 3x}{3}\cdot(2x-5) - \int \frac{-\cos 3x}{3}\cdot 2\,dx = -\frac{2x-5}{3}\cdot\cos 3x + \frac{2}{3}\int\cos 3x\,dx =$

$= -\frac{2x-5}{3}\cdot\cos 3x + \frac{2}{3}\cdot\frac{\sin 3x}{3} + c;\ c\in\mathbb{R}$

d) $\int (5x+2)\cdot 2^x\,dx = \left| \begin{array}{ll} f' = 2^x & g = 5x+2 \\ f = \frac{2^x}{\ln 2} & g' = 5 \end{array}\right| = (5x+2)\cdot\frac{2^x}{\ln 2} - 5\int\frac{2^x}{\ln 2}\,dx = (5x+2)\cdot\frac{2^x}{\ln 2} - \frac{5}{\ln 2}\cdot\frac{2^x}{\ln 2} + c =$

$= \frac{2^x}{\ln 2}\cdot\left(5x+2-\frac{5}{\ln 2}\right) + c;\ c\in\mathbb{R}$

e) $\int \frac{x}{\sin^2 x}\,dx = \left| \begin{array}{ll} f' = \frac{1}{\sin^2 x} & g = x \\ f = -\cot x & g' = 1 \end{array}\right| = -x\cot x + \int \cot x\,dx = -x\cot x + \ln|\sin x| + c;\ c\in\mathbb{R}$

f) $\int (x^2+6x-7)\cdot\cos x\,dx = \left| \begin{array}{ll} f' = \cos x & g = x^2+6x-7 \\ f = \sin x & g' = 2x+6 \end{array}\right| = (x^2+6x-7)\sin x - \int (2x+6)\sin x\,dx = (x^2+6x-7)\sin x + (2x+6)\cos x - 2\sin x +$

$+ c;\ c\in\mathbb{R}$

$\int (2x+6)\sin x\,dx = \left| \begin{array}{ll} f' = \sin x & g = 2x+6 \\ f = -\cos x & g' = 2 \end{array}\right| = -(2x+6)\cos x + \int 2\cos x\,dx = -(2x+6)\cos x + 2\sin x + c$

3.3.1

g) $\int (4x-x^2)\cdot 5^x\,dx = \begin{vmatrix} f' = 5^x & g = 4x-x^2 \\ f = \frac{5^x}{\ln 5} & g' = 4-2x \end{vmatrix} = \frac{5^x}{\ln 5}\cdot(4x-x^2) - \int \frac{5^x}{\ln 5}\cdot(4-2x) = \frac{5^x}{\ln 5}\cdot(4x-x^2) - \frac{1}{\ln 5}\cdot\int 5^x\cdot(4-2x)\,dx =$

$= \frac{5^x}{\ln 5}\cdot(4x-x^2) - \frac{1}{\ln 5}\cdot\left[\frac{5^x}{\ln 5}\cdot(4-2x) + \frac{2}{\ln^2 5}\cdot 5^x\right] = \frac{5^x}{\ln 5}\cdot\left(4x-x^2 - \frac{4-2x}{\ln 5} - \frac{2}{\ln^2 5}\right) + c;\ c\in R$

$\int 5^x\cdot(4-2x)\,dx = \begin{vmatrix} f' = 5^x & g = 4-2x \\ f = \frac{5^x}{\ln 5} & g' = -2 \end{vmatrix} = (4-2x)\cdot\frac{5^x}{\ln 5} + 2\cdot\int\frac{5^x}{\ln 5}\,dx = \frac{5^x}{\ln 5}\cdot(4-2x) + \frac{2}{\ln 5}\cdot\frac{5^x}{\ln 5} + c$

h) $\int (x^2+2x-3)\cdot e^{-x}\,dx = \begin{vmatrix} f' = e^{-x} & g = x^2+2x-3 \\ f = -e^{-x} & g' = 2x+2 \end{vmatrix} = -e^{-x}\cdot(x^2+2x-3) + \int e^{-x}\cdot(2x+2)\,dx = -e^{-x}\cdot(x^2+2x-3+2x+4) =$

$= -e^{-x}\cdot(x^2+4x+1) + c;\ c\in R$

$\int (2x+2)e^{-x}\,dx = \begin{vmatrix} f' = e^{-x} & g = 2x+2 \\ f = -e^{-x} & g' = 2 \end{vmatrix} = -e^{-x}\cdot(2x+2) + 2\cdot\int e^{-x}\,dx = -e^{-x}\cdot(2x+2+2) = -e^{-x}\cdot(2x+4) + c$

i) $\int (3x+5)\cdot\cos\frac{x}{3}\,dx = \begin{vmatrix} f' = \cos\frac{x}{3} & g = 3x+5 \\ f = 3\cdot\sin\frac{x}{3} & g' = 3 \end{vmatrix} = 3\cdot\sin\frac{x}{3}\cdot(3x+5) - 9\cdot\int\sin\frac{x}{3}\,dx = 3\cdot\sin\frac{x}{3}\cdot(3x+5) + 9\cdot\cos\frac{x}{3}\cdot 3 + c =$

$= 3\cdot(3x+5)\cdot\sin\frac{x}{3} + 27\cdot\cos\frac{x}{3} + c;\ c\in R$

derivácia $\left(\cos\frac{x}{3}\right)' = -\sin\frac{x}{3}\cdot\frac{1}{3}$

integrál a derivácia sú navzájom inverzné (opačné) funkcie,

preto integrál $\int\cos\frac{x}{3}\,dx$ nebude $\sin\frac{x}{3}\cdot\frac{1}{3}$, ale $\sin\frac{x}{3}\cdot 3$, tak aby platilo: $\left(3\cdot\sin\frac{x}{3}\right)' = \cos\frac{x}{3}$

3.3.1

j) $\int (2x-7)\cdot \mathrm{tg}^2 x\,dx = \left| \begin{matrix} f' = \mathrm{tg}^2 x & g = 2x-7 \\ f = \mathrm{tg}\,x - x & g' = 2 \end{matrix} \right| = \circledast$

$$\int \mathrm{tg}^2 x\,dx = \int \left(\frac{\sin x}{\cos x}\right)^2 dx = \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos^2 x} - 1\right) dx = \mathrm{tg}\,x - x + c$$

$$\circledast = (\mathrm{tg}\,x - x)\cdot(2x-7) - 2\cdot\int(\mathrm{tg}\,x - x)\,dx = (\mathrm{tg}\,x - x)(2x-7) - 2\cdot\int\left(\frac{\sin x}{\cos x} - x\right)dx = (\mathrm{tg}\,x - x)(2x-7) - 2\cdot\left[-\ln|\cos x| - \frac{x^2}{2}\right] + c =$$

$$= (\mathrm{tg}\,x - x)\cdot(2x-7) + 2\cdot\ln|\cos x| + x^2 + c;\ c\in\mathbb{R}$$

3.3.2

a) $\int x^3 \cdot \ln x\,dx = \left| \begin{matrix} f' = x^3 & g = \ln x \\ f = \frac{x^4}{4} & g' = \frac{1}{x} \end{matrix} \right| = \frac{x^4}{4}\cdot\ln x - \frac{1}{4}\cdot\int\frac{x^4}{x}\,dx = \frac{x^4}{4}\cdot\ln x - \frac{1}{4}\int x^3\,dx = \frac{x^4}{4}\cdot\ln x - \frac{1}{4}\cdot\frac{x^4}{4} + c =$

$$= \frac{x^4}{4}\cdot\left(\ln x - \frac{1}{4}\right) + c;\ c\in\mathbb{R}$$

b) $\int \mathrm{arccotg}\,x\,dx = \int (1\cdot \mathrm{arccotg}\,x)\,dx = \left| \begin{matrix} f' = 1 & g = \mathrm{arccotg}\,x \\ f = x & g' = -\frac{1}{1+x^2} \end{matrix} \right| = x\cdot\mathrm{arccotg}\,x + \int\frac{x}{1+x^2}\,dx =$

$$= x\cdot\mathrm{arccotg}\,x + \frac{1}{2}\int\frac{2x}{1+x^2}\,dx =$$

$$= x\cdot\mathrm{arccotg}\,x + \frac{1}{2}\cdot\ln|1+x^2| + c;\ c\in\mathbb{R}$$

3.3.2

c) $$\int \arccos x \, dx = \left| \begin{array}{ll} f' = 1 & g = \arccos x \\ f = x & g' = \dfrac{-1}{\sqrt{1-x^2}} \end{array} \right| = x \cdot \arccos x + \int \frac{x}{\sqrt{1-x^2}} \, dx = x \cdot \arccos x - \sqrt{1-x^2} + c; \ c \in \mathbb{R}$$

$$\int \frac{x}{\sqrt{1-x^2}} \, dx = \left| \begin{array}{l} \text{subst.:} \\ 1-x^2 = t \\ -2x \, dx = dt \end{array} \right| = -\frac{1}{2} \int \frac{1}{\sqrt{t}} \, dt = -\frac{1}{2} \int t^{-\frac{1}{2}} \, dt = -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = -\sqrt{1-x^2} + c$$

d) $$\int \operatorname{arccotg} 2x \, dx = \left| \begin{array}{ll} f' = 1 & g = \operatorname{arccotg} 2x \\ f = x & g' = -\dfrac{1}{1+(2x)^2} \cdot 2 \end{array} \right| = x \cdot \operatorname{arccotg} 2x + 2 \int x \cdot \frac{1}{1+4x^2} \, dx = x \cdot \operatorname{arccotg} 2x + \frac{1}{4} \ln |1+4x^2| + c$$

$$\int \frac{x}{1+4x^2} \, dx = \frac{1}{8} \int \frac{8x}{1+4x^2} \, dx = \frac{1}{8} \cdot \ln |1+4x^2| + c$$

e) $$\int x \cdot \ln^2 x \, dx = \left| \begin{array}{ll} f' = x & g = \ln^2 x \\ f = \dfrac{x^2}{2} & g' = 2 \cdot \ln x \cdot \dfrac{1}{x} \end{array} \right| = \frac{x^2}{2} \cdot \ln^2 x - \int \frac{x^2}{2} \cdot \frac{2}{x} \cdot \ln x \, dx = \frac{x^2}{2} \ln^2 x - \int x \cdot \ln x \, dx = \circledast$$

$$\int x \cdot \ln x \, dx = \left| \begin{array}{ll} f' = x & g = \ln x \\ f = \dfrac{x^2}{2} & g' = \dfrac{1}{x} \end{array} \right| = \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx = \frac{x^2}{2} \ln x - \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$\circledast = \frac{x^2}{2} \ln^2 x - \left(\frac{x^2}{2} \ln x - \frac{1}{4} x^2 \right) = \frac{x^2}{2} \ln^2 x - \frac{x^2}{2} \ln x + \frac{1}{4} x^2 = \frac{x^2}{2} \cdot \left(\ln^2 x - \ln x + \frac{1}{2} \right) + c; \ c \in \mathbb{R}$$

f) $\int \frac{\ln x}{x^3}\,dx = \left| \begin{matrix} f' = x^{-3} & g = \ln x \\ f = \frac{x^{-2}}{-2} & g' = \frac{1}{x} \end{matrix} \right| = \frac{-\ln x}{2x^2} - \int \frac{-1}{2x^2}\cdot\frac{1}{x}\,dx = \frac{-\ln x}{2x^2} + \frac{1}{2}\cdot\int x^{-3}\,dx =$

$= -\frac{\ln x}{2x^2} + \frac{1}{2}\cdot\frac{x^{-2}}{-2} + c = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c = -\frac{1}{2x^2}\cdot\left(\ln x + \frac{1}{2}\right) + c;\ c\in\mathbb{R}$

h) $\int x^2\cdot \operatorname{arctg} x\,dx = \left| \begin{matrix} f' = x^2 & g = \operatorname{arctg} x \\ f = \frac{x^3}{3} & g' = \frac{1}{1+x^2} \end{matrix} \right| = \frac{x^3}{3}\cdot \operatorname{arctg} x - \frac{1}{3}\int \frac{x^3}{1+x^2}\,dx = \circledast$

$\int \frac{x^3}{1+x^2}\,dx = \frac{1}{2}\int \frac{x^2\cdot 2x}{1+x^2}\,dx = \left| \begin{matrix} 1+x^2 = t \\ 2x\,dx = dt \\ x^2 = t-1 \end{matrix} \right| = \frac{1}{2}\int \frac{t-1}{t}\,dt = \frac{1}{2}\cdot\left(\int 1\,dt - \int t^{-1}\,dt\right) =$

$= \frac{1}{2}\cdot\left(t - \ln|t|\right) + c = \frac{1}{2}\cdot(1+x^2) - \frac{1}{2}\ln|1+x^2| + c$

$\circledast = \frac{x^3}{3}\operatorname{arctg} x - \frac{1}{3}\cdot\left[\frac{1}{2}(1+x^2) - \frac{1}{2}\ln|1+x^2|\right] = \frac{x^3}{3}\operatorname{arctg} x - \frac{1}{6}\cdot(1+x^2) + \frac{1}{6}\ln|1+x^2| + c;\ c\in\mathbb{R}$