8.1. Nájdite lokálne extrémy funkcie f, ak existujú.

a) $f: y = x^2 - 6x + 1$

b) $f: y = 3 + 10x - 5x^2$

c) $f: y = x^3 - 3x^2 - 9x + 7$

d) $f: y = x - \frac{16}{3}x^3$

e) $f: y = (x+2)^2(x+5)$

f) $f: y = -(1-x)(x-3)^2$

g) $f: y = -(x+1)^2(x-3)^2$

h) $f: y = (2x+1)^2(2x-1)^2$

i) $f: y = \frac{x^4}{4} - \frac{x^3}{3}$

j) $f: y = x^4 + 2x^3 - 3$

k) $f: y = 1 - \frac{x^5}{5} - \frac{x^4}{4}$

l) $f: y = x^5 + x^3 + 1$

8.2. Nájdite lokálne extrémy funkcie f, ak existujú.

a) $f: y = \frac{x}{2} + \frac{2}{x}$

b) $f: y = \frac{x^2}{x+3}$

c) $f: y = \frac{2x+1}{x^2}$

d) $f: y = \frac{x}{1+x^2}$

e) $f: y = \frac{1+x^2}{1-x^2}$

f) $f: y = 1 + \frac{1}{x^2 - x}$

g) $f: y = \frac{x^3}{x^2+1}$

h) $f: y = \frac{x^4+1}{x^2}$

i) $f: y = \frac{1}{x^4-1}$

j) $f: y = \frac{(x+1)^2}{x^2-2x}$

8.3. Nájdite lokálne extrémy funkcie f, ak existujú.

a) $f: y = x - \sqrt{x-1}$

b) $f: y = 2x + \sqrt{2x-1}$

c) $f: y = 4 - \sqrt[3]{x}$

d) $f: y = 1 - \sqrt[3]{x^2}$

e) $f: y = 2 - \sqrt[3]{(2-x)^2}$

f) $f: y = 3 \cdot \sqrt[3]{(x+1)^2} - 2x$

g) $f: y = x \cdot \sqrt{9-x}$

h) $f: y = (5-x) \cdot \sqrt[3]{x^2}$

8.4. Nájdite lokálne extrémy funkcie f, ak existujú.

a) $f: y = x \cdot e^{-x}$

b) $f: y = (4-x)e^{4-x}$

c) $f: y = (x^2+1) \cdot e^{-x}$

d) $f: y = e^{-x^2}$

e) $f: y = x \cdot e^{-\frac{x^2}{2}}$

f) $f: y = x + e^{-x}$

g) $f: y = e^x + e^{-x}$

h) $f: y = \frac{e^x}{x+1}$

9.1. Nájdite lokálne extrémy funkcie f, ak existujú.

a) $f: y = x \cdot \ln x$

b) $f: y = \frac{x^2}{2} - \ln x$

c) $f: y = \frac{1}{x} + \ln x$

d) $f: y = \frac{1 + \ln x}{x}$

e) $f: y = \ln(4x - x^2)$

f) $f: y = \ln^2 x - 2\ln x$

g) $f: y = \ln\left(\frac{x-1}{x+1}\right)$

h) $f: y = \frac{\ln x}{x^2}$

i) $f: y = \frac{\ln x}{\sqrt{2x}}$

j) $f: y = \frac{\ln^2 x}{x}$

9.2. Celkové náklady na výrobu x jednotiek určitého tovaru sú dané funkciou $C(x) = 0{,}01x^3 + 10x + 160$. Aký veľký musí byť objem produkcie, aby priemerné náklady boli najmenšie.

9.3. Pre danú funkciu dopytu a funkcie celkových nákladov zistite množstvo výrobkov, ktoré sa musia vyrábať a predávať, aby sa dosiahol maximálny zisk. Určite hodnotu maximálneho zisku a predajnú cenu, ktorá tento zisk maximalizuje.

a) $p(x) = 18 - 2x$

$C(x) = 3 + 6x$

b) $p(x) = 45 - 3x$

$C(x) = 5x + x^2$

c) $p(x) = 30 - \frac{27}{2}x$

$C(x) = 3 + 6x - x^3$

d) $p(x) = 60 - 27x$

$C(x) = 6 + 12x - 2x^3$

9.4. Pre každú funkciu dopytu v nasledujúcich úlohách určte množstvo produkcie, pre ktoré sa dosahujú maximálne príjmy a predajnú cenu, ktorá je výsledkom týchto príjmov.

a) $p(x) = 21 - 0{,}7x$

b) $p(x) = 24 - 0{,}5x^2$

c) $p(x) = 6e^{-0{,}02x}$

d) $p(x) = \sqrt{243 - 9x}$

9.5. Je daná funkcia dopytu $p(x) = 45 - 0{,}5x$ a funkcia priemerných nákladov $\tilde{C}(x) = x^2 - 8x + 57 + \frac{2}{x}$. Určte množstvo

a) predanej produkcie v hl, ktoré maximalizujú príjmy;

b) vyrobenej produkcie v hl, ktoré minimalizujú marginálne náklady;

c) vyrobenej a predanej produkcie v hl, ktoré maximalizuje celkový zisk.

Teória: Aplikácie derivácie funkcie:

1. Monotónnosť: pre všetky $x \in D(f)$ také, že $f'(x) > 0$ platí, že funkcia rastie

pre $\forall x \in (a,b)$: $f'(x) < 0 \Rightarrow f(x)$ na (a,b) klesá

2. Konvexnosť, konkávnosť: pre $\forall x \in (a,b)$: $f''(x) > 0 \Rightarrow f(x)$ je na (a,b) konvexná

pre $\forall x \in (a,b)$: $f''(x) < 0 \Rightarrow f(x)$ je na (a,b) konkávna

3. Lokálne extrémy: pre $\forall x_0 \in D(f)$: $f'(x_0) = 0 \Rightarrow f(x)$ má v bode x_0 lokálny extrém

x_0 sa nazýva stacionárny bod

ak $f''(x_0) > 0 \Rightarrow f(x)$ má v x_0 lokálne minimum

ak $f''(x_0) < 0 \Rightarrow f(x)$ má v x_0 lokálne maximum

4. Inflexné body: pre $\forall x_0 \in D(f)$: $f''(x_0) = 0 \Rightarrow f(x)$ sa mení v x_0 z konvexnej na konkávnu (alebo opačne)

x_0 sa nazýva inflexný bod

7.2

m) $\lim\limits_{x\to 0} \dfrac{x-\sin x}{e^x-e^{-x}-2x} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{1-\cos x}{e^x+e^{-x}-2} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{0} \dfrac{\sin x}{e^x-e^{-x}} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{0} \dfrac{\cos x}{e^x+e^{-x}} = \underline{\underline{\dfrac{1}{2}}}$

o) $\lim\limits_{x\to 0} \dfrac{e^{3x}-3x-1}{\sin^2 x} \overset{LH}{=} \lim\limits_{0} \dfrac{3e^{3x}-3}{2\sin x\cdot\cos x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{3\cdot 3\, e^{3x}}{2\cdot(\cos^2 x-\sin^2 x)} = \underline{\underline{\dfrac{9}{2}}}$

p) $\lim\limits_{x\to 0} \dfrac{1-\cos 3x}{\sin^2 x} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\sin 3x\cdot 3}{2\sin x\cos x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{3\cos 3x\cdot 3}{2(\cos^2 x-\sin^2 x)} = \underline{\underline{\dfrac{9}{2}}}$

q) $\lim\limits_{x\to 0} \dfrac{1-\cos 2x}{e^{x^2}-1} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\sin 2x\cdot 2}{e^{x^2}\cdot 2x} = \lim\limits_{x\to 0} \dfrac{\sin 2x}{x\cdot e^{x^2}} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\cos 2x\cdot 2}{e^{x^2}+xe^{x^2}\cdot 2x} = \lim\limits_{x\to 0} \dfrac{2\cos 2x}{e^{x^2}(1+2x^2)} = \dfrac{2}{1} = \underline{\underline{2}}$

r) $\lim\limits_{x\to 0} \dfrac{x^3}{x-\operatorname{arctg} x} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{3x^2}{1-\frac{1}{1+x^2}} = \lim\limits_{0} \dfrac{3x^2}{\frac{1+x^2-1}{1+x^2}} = \lim\limits_{x\to 0} \dfrac{3\cancel{x^2}(1+x^2)}{\cancel{x^2}} = \lim\limits_{x\to 0} 3\cdot(1+x^2) = \underline{\underline{3}}$

8.1

a) $f: y = x^2-6x+1$

$y' = 2x-6 = 0$ → položíme rovné 0 a zistíme, pre ktoré x platí rovnosť

$2x = 6$

$\boxed{x_0 = 3}$

$y'' = 2 > 0 \Rightarrow$ v $x_0 = 3$ je stacionárny bod a keďže 2. derivácia je > 0, v bode $\underline{x_0 = 3}$ má funkcia lokálne minimum

8.1

b) $f: y = 3 + 10x - 5x^2$

$y' = 10 - 10x = 0$

$\boxed{x_0 = 1}$

$y'' = (10-10x)' = -10 < 0 \Rightarrow$ v bode $x_0 = 1$ má funkcia lokálne maximum

c) $f: y = x^3 - 3x^2 - 9x + 7$

$y' = 3x^2 - 6x - 9 = 0$

$D = 36 + 4 \cdot 3 \cdot 9 = 144$

$x_{1,2} = \dfrac{6 \pm 12}{6} = \begin{cases} 3 \\ -1 \end{cases}$

$y'' = 6x - 6$

$y''(3) = 6 \cdot 3 - 6 = 12 > 0 \Rightarrow$ v $x_0 = 3$ má funkcia lok. minimum

$y''(-1) = 6 \cdot (-1) - 6 = -12 < 0 \Rightarrow$ v $x_0 = -1$ má funkcia lok. maximum

d) $f: y = x - \frac{16}{3}x^3$

$y' = 1 - \frac{16}{3} \cdot 3x^2 = 1 - 16x^2 = 0$

$16x^2 = 1$

$x^2 = \frac{1}{16}$

$x = \pm \frac{1}{4}$

$y'' = -16 \cdot 2x = -32x$

$y''(-\frac{1}{4}) = -16 \cdot 2 \cdot (-\frac{1}{4}) = 8 > 0 \Rightarrow$ v $x_0 = -\frac{1}{4}$ lok. minimum

$y''(\frac{1}{4}) = -32 \cdot \frac{1}{4} = -8 < 0 \Rightarrow$ v $x_0 = \frac{1}{4}$ lok. maximum

e) $f: y = (x+2)^2(x+5)$

$y' = 2(x+2)(x+5) + (x+2)^2 = (x+2)(2x+10+x+2) = (x+2)(3x+12) =$

$= 3(x+2)(x+4) = 0 \Leftrightarrow x_0 = -2 \wedge x_0 = -4$

$y'' = 3(x+4) + 3(x+2) = 3x + 12 + 3x + 6 = 6x + 18$

$y''(-2) = 6 \cdot (-2) + 18 = 6 > 0 \Rightarrow$ v $x_0 = -2$ lok. minimum

$y''(-4) = 6 \cdot (-4) + 18 = -6 < 0 \Rightarrow$ v $x_0 = -4$ lok. maximum

8.1

f) $f: y = -(1-x)(x-3)^2 = (x-1)(x-3)^2$

$$y' = (x-3)^2 + (x-1)\cdot 2(x-3) = (x-3)(x-3+2x-2) = (x-3)(3x-5) = 0 \iff x_0 = 3 \wedge x_0 = \frac{5}{3}$$

$$y'' = (3x-5) + (x-3)\cdot 3 = 3x-5+3x-9 = 6x-14$$

$y''(3) = 6\cdot 3 - 14 = 4 > 0 \Rightarrow$ v $x_0 = 3$ lok. minimum

$y''(\frac{5}{3}) = 6\cdot\frac{5}{3} - 14 = -4 < 0 \Rightarrow$ v $x_0 = \frac{5}{3}$ lok. maximum

g) $f: y = -(x+1)^2(x-3)^2$

$$y' = -2(x+1)(x-3)^2 - (x+1)^2\cdot 2(x-3) = (x+1)(x-3)\cdot[-2(x-3)-2(x+1)] = (x+1)(x-3)(-2x+6-2x-2) =$$

$$= (x+1)(x-3)(-4x+4) = 4\cdot(x+1)(x-3)(1-x) = 0 \iff x_0 = -1 \wedge x_0 = 3 \wedge x_0 = 1$$

$$y'' = [4(x+1)(x-3)(1-x)]' = [4(x-3)\cdot(1+x)(1-x)]' = [4(x-3)\cdot(1-x^2)]' = 4(1-x^2) + 4(x-3)\cdot(-2x) = 4 - 4x^2 - 8\cdot(x^2-3x) =$$

$$= 4 - 4x^2 - 8x^2 + 24x = -12x^2 + 24x + 4 = -4\cdot(3x^2 - 6x - 1)$$

$y''(-1) = -4\cdot(3\cdot 1 + 6\cdot 1 - 1) = -4\cdot 8 = -32 \Rightarrow$ v $x_0 = -1$ je lok. maximum

$y''(3) = -4\cdot(3\cdot 9 - 6\cdot 3 - 1) = -4\cdot 8 = -32 \Rightarrow$ v $x_0 = 3$ je lok. maximum

$y''(1) = -4\cdot(3\cdot 1 - 6\cdot 1 - 1) = -4\cdot(-4) = 16 \Rightarrow$ v $x_0 = 1$ je lok. minimum

8.1

h) $f: y = (2x+1)^2 \cdot (2x-1)^2$

$y' = 2(2x+1)\cdot 2 \cdot (2x-1)^2 + (2x+1)^2 \cdot 2 \cdot (2x-1)\cdot 2 = (2x+1)(2x-1)[4(2x-1)+4(2x+1)] = (2x+1)(2x-1)(8x-4+8x+4) =$

$= (2x+1)(2x-1)\cdot 16x = 0 \Leftrightarrow \underline{x_0 = -\frac{1}{2} \wedge x_0 = \frac{1}{2} \wedge x_0 = 0}$

$y'' = [16(2x+1)(2x-1)x]' = [16x\cdot(4x^2-1)]' = 16(4x^2-1) + 16x\cdot(8x) = 16\cdot(4x^2-1+8x^2) = 16\cdot(12x^2-1)$

$y''(-\frac{1}{2}) = 16\cdot(12\cdot\frac{1}{4}-1) = 32 > 0 \Rightarrow$ v $x_0 = -\frac{1}{2}$ lok. minimum

$y''(\frac{1}{2}) = 16\cdot(12\cdot\frac{1}{4}-1) = 32 > 0 \Rightarrow$ v $x_0 = \frac{1}{2}$ lok. minimum

$y''(0) = 16\cdot(12\cdot 0-1) = -16 < 0 \Rightarrow$ v $x_0 = 0$ lok. maximum

i) $f: y = \frac{1}{4}x^4 - \frac{1}{3}x^3$

$y' = \frac{1}{4}\cdot 4x^3 - \frac{1}{3}\cdot 3x^2 = x^3 - x^2 = x^2(x-1) = 0 \Leftrightarrow \underline{x_0 = 0 \wedge x_0 = 1}$

$y'' = 3x^2 - 2x$

$y''(0) = 3\cdot 0 - 2\cdot 0 = 0$

$y''(1) = 3\cdot 1 - 2\cdot 1 = 1 > 0 \Rightarrow$ v $x_0 = 1$ má lok. minimum

8.1

j) $f: y = x^4 + 2x^3 - 3$

$y' = 4x^3 + 6x^2 = 2x^2(2x+3) = 0 \Leftrightarrow x_0 = 0 \wedge x_0 = -\frac{3}{2}$

$y'' = 4x(2x+3) + 2x^2 \cdot 2 = 8x^2 + 12x + 4x^2 = 12x^2 + 12x = 12x(x+1)$

$y''(0) = 0$

$y''(-\frac{3}{2}) = 12 \cdot (-\frac{3}{2}) \cdot (-\frac{3}{2} + 1) = -18 \cdot (-\frac{1}{2}) = 9 > 0 \Rightarrow$ v $x_0 = -\frac{3}{2}$ je lok. minimum

k) $f: y = 1 - \frac{1}{5}x^5 - \frac{1}{4}x^4$

$y' = -x^4 - x^3 = -x^3 \cdot (x+1) = 0 \Leftrightarrow x_0 = 0 \wedge x_0 = -1$

$y'' = -3x^2 \cdot (x+1) - x^3 = -3x^3 - 3x^2 - x^3 = -4x^3 - 3x^2 = -x^2 \cdot (4x+3)$

$y''(0) = 0$

$y''(-1) = -1 \cdot (-4+3) = 1 > 0 \Rightarrow$ v $x_0 = -1$ je lok. minimum

l) $f: y = x^5 + x^3 + 1$

$y' = 5x^4 + 3x^2 = x^2(5x^2+3) = 5x^2 \cdot (x^2 + \frac{3}{5}) = 0 \Leftrightarrow x_0 = 0 \vee x^2 = -\frac{3}{5}$

$\emptyset$

$y'' = 10x(x^2 + \frac{3}{5}) + 5x^2 \cdot 2x = 10x^3 + 6x + 10x^3 = 20x^3 + 6x$

$y''(0) = 0 \Rightarrow$ nemá lokálne extrémy

8.2 a) $f: y = \frac{x}{2} + \frac{2}{x} = \frac{1}{2}x + 2x^{-1}$

$$y' = \frac{1}{2} - 2x^{-2} = \frac{1}{2} - \frac{2}{x^2} = \frac{x^2-4}{2x^2} = 0 \Leftrightarrow x^2 = 4 \Leftrightarrow \underline{x_0 = 2 \wedge x_0 = -2}$$

$$y'' = \frac{2x(2x^2) - (x^2-4)\cdot 4x}{(2x^2)^2} = \frac{4x^3 - 4x^3 + 16x}{4x^4} = \frac{16x}{4x^4} = \frac{4}{x^3}$$

$y''(2) = \frac{4}{2^3} = \frac{1}{2} > 0 \Rightarrow$ v $x_0 = 2$ je lok. minimum

$y''(-2) = \frac{4}{(-2)^3} = -\frac{1}{2} < 0 \Rightarrow$ v $x_0 = -2$ je lok. maximum

b) $f: y = \frac{x^2}{x+3}$

$$y' = \frac{2x\cdot(x+3) - x^2\cdot 1}{(x+3)^2} = \frac{2x^2 + 6x - x^2}{(x+3)^2} = \frac{x^2+6x}{(x+3)^2} = 0 \Leftrightarrow x(x+6) = 0 \Leftrightarrow \underline{x_0 = 0 \wedge x_0 = -6}$$

$$y'' = \frac{(2x+6)(x+3)^2 - (x^2+6x)\cdot 2(x+3)}{(x+3)^4} = \frac{(x+3)[2(x+3)^2 - 2x^2 - 12x]}{(x+3)^4} = \frac{2x^2+12x+18-2x^2-12x}{(x+3)^3} = \frac{18}{(x+3)^3}$$

$y''(0) = \frac{18}{3^3} = \frac{2}{3} > 0 \Rightarrow$ ~~v~~ $x_0 = 0$ je lok. minimum

$y''(-6) = \frac{18}{(-3)^3} = -\frac{2}{3} < 0 \Rightarrow$ v $x_0 = -6$ je lok. maximum

8.2

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c) $f: y = \dfrac{2x+1}{x^2}$

$$y' = \frac{2x^2 - (2x+1)\cdot 2x}{x^4} = \frac{2x^2-4x^2-2x}{x^4} = \frac{-2x^2-2x}{x^4} = \frac{-2x-2}{x^3} = 0 \Leftrightarrow -2x-2=0 \Leftrightarrow x_0 = -1$$

$$y'' = \frac{-2(x^3) - (-2x-2)\cdot 3x^2}{x^6} = \frac{-2x^3+3x^2(2x+2)}{x^6} = \frac{-2x^3+6x^3+6x^2}{x^6} = \frac{x^2(4x+6)}{x^6} = \frac{4x+6}{x^4}$$

$y''(-1) = \dfrac{4\cdot(-1)+6}{(-1)^4} = 2 > 0 \Rightarrow$ v $x_0 = -1$ je lok. minimum

d) $f: y = \dfrac{x}{1+x^2}$

$$y' = \frac{1\cdot(1+x^2) - x\cdot 2x}{(1+x^2)^2} = \frac{1+x^2-2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} = 0 \Leftrightarrow (1-x)(1+x) = 0 \Leftrightarrow x_0 = 1 \wedge x_0 = -1$$

$$y'' = \frac{-2x\cdot(1+x^2)^2 - (1-x^2)\cdot 2(1+x^2)\cdot 2x}{(1+x^2)^4} = \frac{(1+x^2)\cdot[-2x(1+x^2) - (1-x^2)\cdot 4x]}{(1+x^2)^4} = \frac{-2x-2x^3-4x+4x^3}{(1+x^2)^3} =$$

$$= \frac{2x^3-6x}{(1+x^2)^3}$$

$y''(1) = \dfrac{2\cdot 1 - 6\cdot 1}{(1+1)^3} = \dfrac{-4}{8} = -\dfrac{1}{2} < 0 \Rightarrow$ v $x_0 = 1$ je lok. maximum

$y''(-1) = \dfrac{2\cdot(-1) - 6\cdot(-1)}{(1+1)^3} = \dfrac{4}{8} = \dfrac{1}{2} > 0 \Rightarrow$ v $x_0 = -1$ je lok. minimum

8.2

e) $f: y = \frac{1+x^2}{1-x^2}$

$$y' = \frac{2x(1-x^2)-(1+x^2)\cdot(-2x)}{(1-x^2)^2} = \frac{2x-2x^3+2x+2x^3}{(1-x^2)^2} = \frac{4x}{(1-x^2)^2} = 0 \Leftrightarrow \boxed{x_0=0}$$

$$y'' = \frac{4(1-x^2)^2 - 4x\cdot 2(1-x^2)\cdot(-2x)}{(1-x^2)^4} = \frac{(1-x^2)\cdot[4-4x^2+16x^2]}{(1-x^2)^4} = \frac{12x^2+4}{(1-x^2)^3}$$

$$y''(0) = \frac{4}{1} = 4 > 0 \Rightarrow \text{v } x_0=0 \text{ je lok. minimum}$$

f) $f: y = 1 + \frac{1}{x^2-x} = \frac{x^2-x+1}{x^2-x}$

$$y' = \frac{(2x-1)(x^2-x)-(x^2-x+1)(2x-1)}{(x^2-x)^2} = \frac{(2x-1)(x^2-x-x^2+x-1)}{(x^2-x)^2} = \frac{1-2x}{(x^2-x)^2} = 0 \Leftrightarrow 1-2x=0 \Leftrightarrow \boxed{x_0=\frac{1}{2}}$$

$$y'' = \frac{-2(x^2-x)^2-(1-2x)\cdot 2(x^2-x)\cdot(2x-1)}{(x^2-x)^4} = \frac{(x^2-x)\cdot[-2(x^2-x)-2(1-2x)(2x-1)]}{(x^2-x)^4} = \frac{-2x^2+2x-2(2x-1-4x^2+2x)}{(x^2-x)^3} =$$

$$= \frac{-2x^2+2x-4x+2+8x^2-4x}{(x^2-x)^3} = \frac{6x^2-6x+2}{(x^2-x)^3}$$

$$y''\left(\frac{1}{2}\right) = \frac{6\cdot\frac{1}{4}-6\cdot\frac{1}{2}+2}{\left(\frac{1}{4}-\frac{1}{2}\right)^3} = \frac{\frac{3}{2}-3+2}{\left(-\frac{1}{4}\right)^3} = \frac{\frac{1}{2}}{\frac{-1}{64}} = -\frac{64}{2} = -32 < 0 \Rightarrow \text{v } x_0=\frac{1}{2} \text{ je lok. maximum}$$

8.2 g) $f: y = \dfrac{x^3}{x^2+1}$

$$y' = \frac{3x^2(x^2+1) - x^3(2x)}{(x^2+1)^2} = \frac{3x^4+3x^2-2x^4}{(x^2+1)^2} = \frac{x^4+3x^2}{(x^2+1)^2} = \frac{x^2(x^2+3)}{(x^2+1)^2} = 0 \Leftrightarrow \underline{x_0 = 0} \ \vee \ x^2 = -3$$

$$\emptyset$$

$$y'' = \left(\frac{x^4+3x^2}{(x^2+1)^2}\right)' = \frac{(4x^3+6x)(x^2+1)^2 - (x^4+3x^2)\cdot 2\cdot(x^2+1)\cdot 2x}{[(x^2+1)^2]^2} = \frac{(x^2+1)\cdot[(4x^3+6x)(x^2+1) - 4x(x^4+3x^2)]}{(x^2+1)^4} =$$

$$= \frac{4x^5+4x^3+6x^3+6x-4x^5-12x^3}{(x^2+1)^3} = \frac{-2x^3+6x}{(x^2+1)^3}$$

$$y''(0) = \frac{0}{1} = 0 \Rightarrow \underline{\text{funkcia nemá lok. extrémy}}$$

h) $f: y = \dfrac{x^4+1}{x^2}$

$$y' = \frac{4x^3\cdot x^2 - (x^4+1)\cdot 2x}{x^4} = \frac{4x^5-2x^5-2x}{x^4} = \frac{2x(x^4-1)}{x^4} = \frac{2(x^4-1)}{x^3} = 0 \Leftrightarrow x^4 = 1 \Leftrightarrow \underline{x_0=-1 \ \wedge \ x_0 = 1}$$

$$y'' = \frac{2\cdot 4x^3\cdot x^3 - 2(x^4-1)\cdot 3x^2}{x^6} = \frac{8x^6-6x^2(x^4-1)}{x^6} = \frac{8x^6-6x^6+6x^2}{x^6} = \frac{2x^6+6x^2}{x^6} = \frac{2x^4+6}{x^4}$$

$$y''(-1) = \frac{2\cdot 1 + 6}{1} = y''(1) = 8 > 0 \Rightarrow \underline{\text{v } x_0=-1 \text{ aj v } x_0 = 1 \text{ má funkcia lok. minimum}}$$

8.2

i) $f: y = \frac{1}{x^4-1}$

$$y' = \frac{-4x^3}{(x^4-1)^2} = 0 \Leftrightarrow \boxed{x_0 = 0}$$

$$y'' = \frac{-12x^2(x^4-1)^2 + 4x^3 \cdot 2(x^4-1)\cdot 4x^3}{(x^4-1)^4} = \frac{(x^4-1)x^2 \cdot [-12(x^4-1)+32x^4]}{(x^4-1)^4} = \frac{x^2(32x^4-12x^4+12)}{(x^4-1)^3} = \frac{x^2\cdot(20x^4+12)}{(x^4-1)^3}$$

$$y''(0) = \frac{0\cdot(0+12)}{(0-1)^3} = 0 \Rightarrow$$ funkcia nemá lok. extrém

j) $f: y = \frac{(x+1)^2}{x^2-2x}$

$$y' = \frac{2(x+1)(x^2-2x) - (x+1)^2\cdot(2x-2)}{(x^2-2x)^2} = \frac{(x+1)\cdot(2x^2-4x-(2x^2-2))}{(x^2-2x)^2} = \frac{(x+1)(-4x+2)}{(x^2-2x)^2} = 0 \Leftrightarrow \boxed{x_0=-1 \wedge x_0 = \frac{1}{2}}$$

$$y'' = \left(\frac{(x+1)(2-4x)}{(x^2-2x)^2}\right)' = \left(\frac{2x-4x^2+2-4x}{(x^2-2x)^2}\right)' = \left(\frac{-4x^2-2x+2}{(x^2-2x)^2}\right)' = \frac{(-8x-2)(x^2-2x)^2 - (-4x^2-2x+2)\cdot 2(x^2-2x)\cdot(2x-2)}{(x^2-2x)^4} =$$

$$= \frac{(x^2-2x)[(-8x-2)(x^2-2x) - 2(2x-2)(-4x^2-2x+2)]}{(x^2-2x)^4} = \frac{-8x^3+16x^2-2x^2+4x-2(-8x^3-4x^2+4x+8x^2+4x-4)}{(x^2-2x)^3} =$$

$$= \frac{-8x^3+14x^2+4x+16x^3+8x^2-8x-16x^2-8x+8}{(x^2-2x)^3} = \frac{8x^3+6x^2-12x+8}{(x^2-2x)^3}$$

8.2 j)

$$y'' = \frac{8x^3+6x^2-12x+8}{(x^2-2x)^3}$$

$$y''(-1) = \frac{-8+6+12+8}{(1+2)^3} = \frac{18}{27} = \frac{2}{3} > 0 \Rightarrow \text{v } x_0 = -1 \text{ je lok. minimum}$$

$$y''\left(\frac{1}{2}\right) = \frac{8\cdot\frac{1}{8}+6\cdot\frac{1}{4}-12\cdot\frac{1}{2}+8}{\left(\frac{1}{4}-2\cdot\frac{1}{2}\right)^3} = \frac{1+\frac{3}{2}-6+8}{\left(\frac{1}{4}-1\right)^3} = \frac{\frac{9}{2}}{\left(-\frac{3}{4}\right)^3} = \frac{\frac{9}{2}}{-\frac{27}{64}} = -\frac{64\cdot 9}{2\cdot 27} < 0 \Rightarrow \text{v } x_0 = \frac{1}{2} \text{ je lok. maximum}$$

8.3

a) $f: y = x - \sqrt{x-1} = x - (x-1)^{\frac{1}{2}}$

$$y' = 1 - \frac{1}{2}(x-1)^{-\frac{1}{2}} = 1 - \frac{1}{2\sqrt{x-1}} = \frac{2\sqrt{x-1}-1}{2\sqrt{x-1}} = 0 \Leftrightarrow 2\sqrt{x-1} = 1$$

$$\sqrt{x-1} = \frac{1}{2} \quad /^2$$

$$x-1 = \frac{1}{4}$$

$$\boxed{x_0 = \frac{5}{4}}$$

$$y'' = \left[1-\frac{1}{2}(x-1)^{-\frac{1}{2}}\right]' = \frac{1}{4}(x-1)^{-\frac{3}{2}} = \frac{1}{4\sqrt{(x-1)^3}}$$

$$y''\left(\frac{5}{4}\right) = \frac{1}{4\cdot\sqrt{\left(\frac{5}{4}-1\right)^3}} = 2 > 0 \Rightarrow \text{v } x_0 = \frac{5}{4} \text{ je lok. minimum}$$

8.3

b) $f: y = 2x + \sqrt{2x-1} = 2x + (2x-1)^{\frac{1}{2}}$

$y' = 2 + \frac{1}{2}(2x-1)^{-\frac{1}{2}} \cdot 2 = 2 + \frac{1}{\sqrt{2x-1}} = \frac{2\cdot\sqrt{2x-1} + 1}{\sqrt{2x-1}} = 0 \Leftrightarrow \sqrt{2x-1} = -\frac{1}{2} \Rightarrow$ funkcia nemá lok. extrémy

$\emptyset$

c) $f: y = 4 - \sqrt[3]{x} = 4 - x^{\frac{1}{3}}$

$y' = -\frac{1}{3}x^{-\frac{2}{3}} = -\frac{1}{3\cdot\sqrt[3]{x^2}} \neq 0$ nikdy $\Rightarrow$ funkcia nemá lok. extrémy

d) $f: y = 1 - \sqrt[3]{x^2} = 1 - x^{\frac{2}{3}}$

$y' = -\frac{2}{3}x^{-\frac{1}{3}} = \frac{-2}{3\cdot\sqrt[3]{x}} \neq 0$ nikdy $\Rightarrow$ funkcia nemá lok. extrémy

e) $f: y = 2 - \sqrt[3]{(2-x)^2} = 2 - (2-x)^{\frac{2}{3}}$

$y' = \cancel{2} - \frac{2}{3}(2-x)^{-\frac{1}{3}} \cdot (-1) = \frac{2}{3\sqrt[3]{2-x}} \neq 0$ nikdy $\Rightarrow$ funkcia nemá lok. extrémy

f) $f: y = 3\cdot\sqrt[3]{(x+1)^2} - 2x = 3\cdot(x+1)^{\frac{2}{3}} - 2x$

$y' = 3\cdot\frac{2}{3}\cdot(x+1)^{-\frac{1}{3}} - 2 = \frac{2}{\sqrt[3]{x+1}} - 2 = \frac{2 - 2\sqrt[3]{x+1}}{\sqrt[3]{x+1}} = 0 \Leftrightarrow \sqrt[3]{x+1} = 1 \;/^3$

$x+1 = 1$

$x_0 = 0$

8.3

f) $y'' = \left(2(x+1)^{-\frac{1}{3}} - 2\right)' = 2\cdot\left(-\frac{1}{3}\right)(x+1)^{-\frac{4}{3}} = \dfrac{-2}{3\cdot\sqrt[3]{(x+1)^4}}$

$y''(0) = \dfrac{-2}{3\cdot\sqrt[3]{1}} = -\dfrac{2}{3} < 0 \Rightarrow$ v x_0 má funkcia lok. maximum

g) $f: y = x\cdot\sqrt{9-x} = x(9-x)^{\frac{1}{2}}$

$y' = (9-x)^{\frac{1}{2}} + x\cdot\frac{1}{2}(9-x)^{-\frac{1}{2}}\cdot(-1) = \sqrt{9-x} - \frac{x}{2}\cdot\dfrac{1}{\sqrt{9-x}} = \dfrac{2(9-x)-x}{2\sqrt{9-x}} = \dfrac{18-2x-x}{2\sqrt{9-x}} = \dfrac{18-3x}{2\sqrt{9-x}} = 0 \Leftrightarrow \boxed{x_0 = 6}$

$y'' = \left[(9-x)^{\frac{1}{2}} - \frac{x}{2}\cdot(9-x)^{-\frac{1}{2}}\right]' = -\frac{1}{2}(9-x)^{-\frac{1}{2}} - \left(\frac{1}{2}(9-x)^{-\frac{1}{2}} + \frac{x}{2}\cdot\left(-\frac{1}{2}\right)(9-x)^{-\frac{3}{2}}\cdot(-1)\right) = \dfrac{-1}{2\sqrt{9-x}} - \dfrac{1}{2\sqrt{9-x}} - \dfrac{x}{4}\cdot\dfrac{1}{\sqrt{(9-x)^3}} =$

$= \dfrac{-2(9-x)-2(9-x)-x}{4\cdot\sqrt{(9-x)^3}} = \dfrac{-36+3x}{4\cdot\sqrt{(9-x)^3}}$

$y''(6) = \dfrac{-36+18}{4\cdot\sqrt{27}} = \dfrac{-18}{4\sqrt{27}} < 0 \Rightarrow$ v $x_0 = 6$ má funkcia lok. maximum

h) $f: y = (5-x)\cdot\sqrt[3]{x^2} = (5-x)\cdot x^{\frac{2}{3}}$

$y' = (-1)\cdot x^{\frac{2}{3}} + (5-x)\cdot\frac{2}{3}x^{-\frac{1}{3}} = \cancel{\dfrac{-1}{3\sqrt[3]{x^2}} + \dfrac{2(5-x)}{3\sqrt[3]{x}} = \dfrac{-3+(10-2x)\sqrt[3]{x}}{3\sqrt[3]{x^2}}} =$

$-\sqrt[3]{x^2} + \dfrac{2\cdot(5-x)}{3\cdot\sqrt[3]{x}} = \dfrac{-3\cdot\sqrt[3]{x}\cdot\sqrt[3]{x^2} + 10 - 2x}{3\cdot\sqrt[3]{x}} = \dfrac{-3x-2x+10}{3\sqrt[3]{x}} = \dfrac{-5x+10}{3\sqrt[3]{x}} = 0$

$\Leftrightarrow \boxed{x_0 = 2}$

8.3

h) $y'' = \left(\frac{-5x+10}{3\sqrt[3]{x}}\right)' = \left[\frac{1}{3}x^{-\frac{1}{3}}\cdot(10-5x)\right]' = \left(\frac{1}{3}x^{-\frac{1}{3}}(10-5x)\right)' = \frac{1}{3}\cdot\left(-\frac{1}{3}\right)x^{-\frac{4}{3}}\cdot(10-5x) + \frac{1}{3}x^{-\frac{1}{3}}\cdot(-5) =$

$= \frac{10-5x}{-9\cdot\sqrt[3]{x^4}} - \frac{5}{3\sqrt[3]{x}} = \frac{5x-10}{9\cdot\sqrt[3]{x^4}} - \frac{5}{3\cdot\sqrt[3]{x}} = \frac{5x-10-5\cdot(3x)}{9\cdot\sqrt[3]{x^4}} = \frac{-10x-10}{9\cdot\sqrt[3]{x^4}}$

$y''(2) = \frac{-10\cdot 2-10}{9\cdot\sqrt[3]{16}} = \frac{-30}{9\cdot\sqrt[3]{16}} < 0 \Rightarrow$ v $x_0 = 2$ je lok. maximum

8.4

a) $f: y = x \cdot e^{-x}$

$$y' = e^{-x} + x \cdot e^{-x} \cdot (-1) = e^{-x}(1-x) = 0 \Leftrightarrow \boxed{x_0 = 1}$$

$$y'' = e^{-x} \cdot (-1)(1-x) + e^{-x} \cdot (-1) = -e^{-x}(1-x+1) = -e^{-x}(2-x) = e^{-x}(x-2)$$

$$y''(1) = e^{-1} \cdot (-1) = -\frac{1}{e} < 0 \Rightarrow \text{v } x_0 = 1 \text{ je lok. maximum}$$

b) $f: y = (4-x)\, e^{4-x}$

$$y' = (-1) \cdot e^{4-x} + (4-x)\, e^{4-x} \cdot (-1) = e^{4-x}(-1-4+x) = e^{4-x}(x-5) = 0 \Leftrightarrow \boxed{x_0 = 5}$$

$$y'' = e^{4-x} \cdot (-1)(x-5) + e^{4-x} \cdot 1 = e^{4-x} \cdot (-x+5+1) = e^{4-x} \cdot (6-x)$$

$$y''(5) = e^{-1} \cdot 1 = \frac{1}{e} > 0 \Rightarrow \text{v } x_0 = 5 \text{ je lok. minimum}$$

c) $f: y = (x^2+1) \cdot e^{-x}$

$$y' = 2x e^{-x} + (x^2+1) e^{-x} \cdot (-1) = e^{-x}(2x - x^2 - 1) = e^{-x} \cdot (-x^2+2x-1) = 0 \Leftrightarrow -x^2+2x-1 = 0 \;/ \cdot (-1)$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0 \Leftrightarrow \boxed{x_0 = 1}$$

$$y'' = e^{-x} \cdot (-1) \cdot (-x^2+2x-1) + e^{-x} \cdot (-2x+2) = e^{-x} \cdot (x^2-2x+1-2x+2) = e^{-x} \cdot (x^2-4x+3)$$

$$y''(1) = e^{-1} \cdot (1-4+3) = 0 \Rightarrow \text{lok. extrém neexistuje}$$

8.4

d) $f: y = e^{-x^2}$

$y' = e^{-x^2} \cdot (-2x) = -2x e^{-x^2} = 0 \Leftrightarrow \boxed{x_0 = 0}$

$y'' = -2 e^{-x^2} + (-2x) \cdot e^{-x^2} \cdot (-2x) = e^{-x^2} \cdot (-2 + 4x^2) = e^{-x^2} \cdot (4x^2 - 2)$

$y''(0) = e^0 \cdot (-2) = -2 < 0 \Rightarrow$ v $x_0 = 0$ je lok. maximum

e) $f: y = x \cdot e^{\frac{x^2}{2}}$

$y' = e^{\frac{x^2}{2}} + x \cdot e^{\frac{x^2}{2}} \cdot \left(\frac{1}{2} \cdot 2x\right) = e^{\frac{x^2}{2}} \cdot (1 + x^2) \neq 0$ nikdy $\Rightarrow$ funkcia nemá lok. extrém

f) $f: y = x + e^{-x}$

$y' = 1 - e^{-x} = \dfrac{e^x - 1}{e^x} = 0 \Leftrightarrow e^x = 1 \Leftrightarrow \boxed{x_0 = 0}$

$y'' = (1 - e^{-x})' = e^{-x}$

$y''(0) = e^0 = 1 > 0 \Rightarrow$ v $x_0 = 0$ je lok. minimum

g) $f: y = e^x + e^{-x}$

$y' = e^x - e^{-x} = 0 \Leftrightarrow e^x = e^{-x}$

$x = -x \Rightarrow \boxed{x_0 = 0}$

$y'' = e^x + e^{-x}$

$y''(0) = e^0 + e^0 = 2 > 0 \Rightarrow$ v $x_0 = 0$ je lok. minimum

8.4

h) $f: y = \dfrac{e^x}{x+1}$

$$y' = \frac{e^x(x+1) - e^x \cdot 1}{(x+1)^2} = \frac{e^x \cdot x}{(x+1)^2} = 0 \Leftrightarrow \underbrace{x_0 = 0}$$

$$y'' = \frac{(e^x \cdot x + e^x \cdot 1)(x+1)^2 - e^x \cdot x \cdot 2(x+1)}{(x+1)^4} = \frac{e^x(x+1)[(x+1)^2 - 2x]}{(x+1)^4} = \frac{e^x(x+1)(x^2+2x+1-2x)}{(x+1)^4} = \frac{e^x(x^2+1)}{(x+1)^3}$$

$$y''(0) = \frac{e^0 \cdot (0+1)}{(0+1)^3} = \frac{1 \cdot 1}{1} = 1 > 0 \Rightarrow \text{v } x_0 = 0 \text{ je lok. minimum}$$

9.1

~~8.5~~ a) $f: y = x \ln x$

$$y' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1 = 0 \Leftrightarrow \ln x = -1$$

$$e^{\ln x} = e^{-1}$$

$$x = e^{-1} \Rightarrow \underbrace{x_0 = \frac{1}{e}}$$

$$y'' = \frac{1}{x}$$

$$y''\left(\frac{1}{e}\right) = \frac{1}{\frac{1}{e}} = e > 0 \Rightarrow \text{v } x_0 = \frac{1}{e} \text{ je lok. minimum}$$

b) $f: y = \dfrac{x^2}{2} - \ln x$

$$y' = \frac{1}{2} \cdot 2x - \frac{1}{x} = x - \frac{1}{x} = \frac{x^2 - 1}{x} = \frac{(x+1)(x-1)}{x} = 0 \Leftrightarrow \underbrace{x_0 = -1 \wedge x_0 = 1}$$

$$y'' = \left(\frac{x^2-1}{x}\right)' = \frac{2x \cdot x - (x^2-1) \cdot 1}{x^2} = \frac{2x^2 - x^2 + 1}{x^2} = \frac{x^2+1}{x^2}$$

$$\left.\begin{aligned} y''(-1) &= \frac{1+1}{1} = 2 > 0 \\ y''(1) &= \text{— // —} \end{aligned}\right\} \Rightarrow \text{v } x_0 = -1 \text{ a } x_0 = 1 \text{ je lok. minimum}$$

9.1

c) $f: y = \frac{1}{x} + \ln x = x^{-1} + \ln x$

$$y' = -x^{-2} + \frac{1}{x} = \frac{-1+x}{x^2} = 0 \Leftrightarrow \boxed{x_0 = 1}$$

$$y'' = \frac{1 \cdot x^2 - (-1+x) \cdot 2x}{x^4} = \frac{x^2 + 2x - 2x^2}{x^4} = \frac{-x^2 + 2x}{x^4} = \frac{x(2-x)}{x^4} = \frac{2-x}{x^3}$$

$$y''(1) = \frac{2-1}{1} = 1 > 0 \Rightarrow \text{v } x_0 = 1 \text{ je lok. minimum}$$

d) $f: y = \frac{1+\ln x}{x}$

$$y' = \frac{\frac{1}{x} \cdot x - (1+\ln x) \cdot 1}{x^2} = \frac{1-1-\ln x}{x^2} = \frac{-\ln x}{x^2} = 0 \Leftrightarrow -\ln x = 0 \quad / \cdot (-1)$$

$$\ln x = 0$$

$$e^{\ln x} = e^0 \Leftrightarrow \boxed{x_0 = 1}$$

$$y'' = \frac{\left(-\frac{1}{x}\right) x^2 - (-\ln x) \cdot 2x}{x^4} = \frac{-x + 2x\ln x}{x^4} = \frac{x(2\ln x - 1)}{x^4} = \frac{2\ln x - 1}{x^3}$$

$$y''(1) = \frac{2 \cdot \ln 1 - 1}{1} = \frac{0-1}{1} = -1 < 0 \Rightarrow \text{v } x_0 = 1 \text{ je lok. maximum}$$

e) $f: y = \ln(4x - x^2)$

$$y' = \frac{1}{4x - x^2} \cdot (4-2x) = \frac{4-2x}{4x-x^2} = 0 \Leftrightarrow 4 - 2x = 0 \Leftrightarrow \boxed{x_0 = 2}$$

$$y'' = \frac{-2(4x - x^2) - (4-2x)(4-2x)}{(4x-x^2)^2} = \frac{-8x + 2x^2 - 16 + 16x - 4x^2}{(4x-x^2)^2} = \frac{-2x^2 + 8x - 16}{(4x-x^2)^2}$$

$$y''(2) = \frac{-2 \cdot 4 + 8 \cdot 2 - 16}{(8-4)^2} = \frac{-8}{16} = -\frac{1}{2} < 0 \Rightarrow \text{v } x_0 = 2 \text{ je lok. maximum}$$

9.1

f) $f: y = \ln^2 x - 2\ln x$

$$y' = 2\ln x \cdot \frac{1}{x} - 2\cdot\frac{1}{x} = \frac{2\ln x - 2}{x} = 0 \Leftrightarrow 2\ln x = 2$$
$$\ln x = 1 \Leftrightarrow \underline{x_0 = e}$$

$$y'' = \frac{2\cdot\frac{1}{x}\cdot x - (2\ln x - 2)\cdot 1}{x^2} = \frac{2 - 2\ln x + 2}{x^2} = \frac{4 - 2\ln x}{x^2}$$

$$y''(e) = \frac{4 - 2\cdot\ln e}{e^2} = \frac{4-2}{e^2} = \frac{2}{e^2} > 0 \Rightarrow \text{v } x_0 = e \text{ je lok. minimum}$$

g) $f: y = \ln\left(\frac{x-1}{x+1}\right)$

$$y' = \frac{1}{\frac{x-1}{x+1}} \cdot \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{x+1}{x-1}\cdot\frac{2}{(x+1)^2} = \frac{2}{(x-1)(x+1)} \neq 0 \text{ nikdy} \Rightarrow \text{funkcia nemá lok. extrém}$$

h) $f: y = \frac{\ln x}{x^2}$

$$y' = \frac{\frac{1}{x}\cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x\ln x}{x^4} = \frac{x(1-2\ln x)}{x^4} = \frac{1-2\ln x}{x^3} = 0 \Leftrightarrow 1 = 2\ln x$$
$$\ln x = \frac{1}{2} \Rightarrow \underline{x_0 = \sqrt{e}}$$

$$y'' = \frac{-2\cdot\frac{1}{x}\cdot x^3 - (1-2\ln x)\cdot 3x^2}{x^6} = \frac{-2x^2 - 3x^2 + 6x^2\ln x}{x^6} = \frac{x^2\cdot(-5+6\ln x)}{x^6} = \frac{-5+6\ln x}{x^4}$$

$$y''(\sqrt{e}) = \frac{-5 + 6\cdot\ln e^{\frac{1}{2}}}{(\sqrt{e})^4} = \frac{-5+6\cdot\frac{1}{2}}{e^2} = \frac{-5+3}{e^2} = \frac{-2}{e^2} < 0 \Rightarrow \text{v } x_0 = \sqrt{e} \text{ je lok. maximum}$$

9.1

i) $f: y=\frac{\ln x}{\sqrt{2x}}=\frac{\ln x}{(2x)^{\frac{1}{2}}}$

$$y'=\frac{\frac{1}{x}\cdot(2x)^{\frac{1}{2}}-\ln x\cdot\frac{1}{2}(2x)^{-\frac{1}{2}}\cdot 2}{2x}=\frac{\frac{1}{x}\cdot\sqrt{2x}-\frac{\ln x}{\sqrt{2x}}}{2x}=\frac{\frac{\sqrt{2x}\cdot\sqrt{2x}-x\ln x}{x\sqrt{2x}}}{2x}=\frac{2x-\ln x\cdot x}{2x^2\sqrt{2x}}=\frac{x(2-\ln x)}{2x^2\sqrt{2x}}=$$

$$=\frac{2-\ln x}{2x\sqrt{2x}}=0 \Leftrightarrow 2=\ln x$$

$$e^{\ln x}=e^2 \Leftrightarrow \boxed{x_0=e^2}$$

$$\boxed{\frac{a^2}{a}=a}$$

$$y''=\frac{-\frac{1}{x}\cdot 2x\sqrt{2x}-(2-\ln x)\cdot\left(2\sqrt{2x}+2x\cdot\frac{1}{2}\cdot(2x)^{-\frac{1}{2}}\cdot 2\right)}{\left(2x\sqrt{2x}\right)^2}=\frac{-2\sqrt{2x}+(\ln x-2)\left(2\sqrt{2x}+\frac{2x}{\sqrt{2x}}\right)}{4x^2\cdot 2x}=$$

$$=\frac{-2\sqrt{2x}+(\ln x-2)\left(2\sqrt{2x}+\sqrt{2x}\right)}{8x^3}=\frac{-2\sqrt{2x}+(\ln x-2)\cdot 3\sqrt{2x}}{8x^3}=\frac{\sqrt{2x}\cdot(-2+3\ln x-6)}{8x^3}=$$

$$=\frac{\sqrt{2x}\cdot(3\ln x-8)}{8x^3}$$

$$y''(e^2)=\frac{\sqrt{2e^2}\cdot(3\ln e^2-8)}{8\cdot(e^2)^3}=\frac{\sqrt{2}e\cdot(3\cdot 2-8)}{8e^6}=\frac{\sqrt{2}\cdot(-2)}{8e^5}=\frac{-2\sqrt{2}}{8e^5}=\frac{-\sqrt{2}}{4e^5}<0\Rightarrow \text{v } x_0=e^2 \text{ je lok. max.}$$

j) $f: y=\frac{\ln^2 x}{x}$

$$y'=\frac{2\ln x\cdot\frac{1}{x}\cdot x-\ln^2 x\cdot 1}{x^2}=\frac{2\ln x-\ln^2 x}{x^2}=\frac{\ln x\cdot(2-\ln x)}{x^2}=0 \Leftrightarrow \ln x=0 \lor \ln x=2$$

$$\boxed{x_0=1 \lor x_0=e^2}$$

$$y''=\left(\frac{2\ln x-\ln^2 x}{x^2}\right)'=\frac{\left(2\cdot\frac{1}{x}-2\ln x\cdot\frac{1}{x}\right)x^2-(2\ln x-\ln^2 x)\cdot 2x}{x^4}=\circledast$$

9.1

j) $\circledast = \dfrac{\frac{2-2\ln x}{x}\cdot x^2 - 2x(2\ln x - \ln^2 x)}{x^4} = \dfrac{x\cdot[2-2\ln x - 4\ln x + 2\ln^2 x]}{x^4} = \dfrac{2\ln^2 x - 6\ln x + 2}{x^3}$

$y''(1) = \dfrac{2\cdot \ln^2 1 - 6\cdot \ln 1 + 2}{1} = \dfrac{2}{1} = 2 > 0 \Rightarrow$ v $x_0 = 1$ je lok. minimum

$y''(e^2) = \dfrac{2\ln^2 e^2 - 6\ln e^2 + 2}{(e^2)^3} = \dfrac{2\cdot 4 - 6\cdot 2 + 2}{e^6} = \dfrac{-2}{e^6} < 0 \Rightarrow$ v $x_0 = e^2$ je lok. maximum

3.1.1

a) $\int (5x^2 - 4x + 10)\,dx = 5\cdot\frac{x^3}{3} - 4\cdot\frac{x^2}{2} + 10x = \frac{5}{3}x^3 - 2x^2 + 10x + c;\ c\in R$

b) $\int x\cdot(3x-4)^2\,dx = \int x\cdot(9x^2 - 24x + 16)\,dx = \int (9x^3 - 24x^2 + 16x)\,dx = 9\cdot\frac{x^4}{4} - 24\cdot\frac{x^3}{3} + 16\cdot\frac{x^2}{2} = \frac{9}{4}x^4 - 8x^3 + 8x^2 + c;\ c\in R$

c) $\int (x^2 - 4x\sqrt[3]{x} + 10\sqrt[4]{x^3})\,dx = \int \left(x^2 - 4x\cdot x^{\frac{1}{3}} + 10\cdot x^{\frac{3}{4}}\right)dx = \int \left(x^2 - 4x^{\frac{4}{3}} + 10x^{\frac{3}{4}}\right)dx = \frac{x^3}{3} - 4\cdot\frac{x^{\frac{7}{3}}}{\frac{7}{3}} + 10\cdot\frac{x^{\frac{7}{4}}}{\frac{7}{4}} + c =$

$= \frac{x^3}{3} - 4\cdot\frac{3}{7}\cdot x^{\frac{7}{3}} + 10\cdot\frac{4}{7}\cdot x^{\frac{7}{4}} + c = \frac{1}{3}x^3 - \frac{12}{7}\cdot\sqrt[3]{x^7} + \frac{40}{7}\cdot\sqrt[4]{x^7} + c;\ c\in R$

d) $\int (x-2)(4-x)\,dx = \int (4x - x^2 - 8 + 2x)\,dx = \int (-x^2 + 6x - 8)\,dx = -\frac{x^3}{3} + 6\cdot\frac{x^2}{2} - 8x = -\frac{1}{3}x^3 + 3x^2 - 8x + c;\ c\in R$

e) $\int (x^2-2)^3\,dx = \int (x^2-2)^2\cdot(x^2-2)\,dx = \int (x^4 - 4x^2 + 4)(x^2-2)\,dx = \int (x^6 - 2x^4 - 4x^4 + 8x^2 + 4x^2 - 8)\,dx = \int (x^6 - 6x^4 + 12x^2 - 8)\,dx =$

$= \frac{x^7}{7} - 6\cdot\frac{x^5}{5} + 12\cdot\frac{x^3}{3} - 8x + c = \frac{1}{7}x^7 - \frac{6}{5}x^5 + 4x^3 - 8x + c;\ c\in R$