V úlohách 1.3.5 – 1.3.8 určte definičný obor daných funkcií.

1.3.5 a) $f_1\colon y=\dfrac{\sqrt{x+1}}{x-4}$ d) $f_4\colon y=\dfrac{\sqrt{2x+10}}{16-x^2}$

b) $f_2\colon y=\sqrt{\dfrac{-3}{x^2-5x+4}}$ e) $f_5\colon y=\dfrac{1}{\sqrt{x}}+\sqrt{x^2-5}$

c) $f_3\colon y=\dfrac{-3}{\sqrt{x^2-3x}}$ f) $f_6\colon y=\dfrac{\sqrt{15+2x-x^2}}{8-2x}$

1.3.6 a) $f_1\colon y=4^{\log\left(2x^2-5x-3\right)}$ d) $f_4\colon y=\log x^2+\log\left(4-x^2\right)$

b) $f_2\colon y=\ln\sqrt{\dfrac{3x-1}{x+4}}$ e) $f_5\colon y=\sqrt{1-\log\left(x^2+7x+10\right)}$

c) $f_3\colon y=\dfrac{-1}{\ln\left(2x-x^2\right)}$ f) $f_6\colon y=\dfrac{\sqrt{\ln\left(x-1\right)}}{x-2}$

1.3.7 a) $f_1\colon y=\arcsin\dfrac{2x+4}{x}$ d) $f_4\colon y=\dfrac{1}{x}+\arccos\left(x^2-1\right)$

b) $f_2\colon y=\operatorname{arccotg}\dfrac{x^2}{x^2-2}$ e) $f_5\colon y=\dfrac{x}{\operatorname{arctg}(12-4x)}$

c) $f_3\colon y=\arccos\dfrac{1}{x^2}$ f) $f_6\colon y=\sqrt{\arcsin\left(x-4\right)}$

1.3.8 a) $f_1\colon y=\log\left(1-2x\right)-3\arcsin\dfrac{3x-1}{2}$

b) $f_2\colon y=5\log\left(\dfrac{x+1}{x-5}\right)-\dfrac{\sqrt{5x-10}}{x^2-36}$

c) $f_3\colon y=\arccos\left(3+2x\right)+\sqrt{\dfrac{x-2}{x+3}}$

d) $f_4\colon y=\dfrac{\sqrt{x^2-5x+6}}{\ln\left(2x-5\right)}-\sqrt{5-x}$

e) $f_5\colon y=\dfrac{\sqrt{x^2-x-2}}{\ln x}-4\cdot\arcsin\dfrac{1-2x}{4}$

f) $f_6\colon y=\sqrt{\log\dfrac{5x-x^2}{4}}+\dfrac{1}{\log_2 x-2}$

g) $f_7\colon y=\arccos\dfrac{3}{2x-5}+\ln\left(6+11x-2x^2\right)$

h) $f_8\colon y=\dfrac{14-x}{\ln\left(x^2-4\right)}+\arccos\left(3x+7\right)$

1.3.5 (1a)

a) $f_1: y = \dfrac{\sqrt{x+1}}{x-4}$; $D(f_1)$: $x+1 \geq 0 \ \wedge \ x-4 \neq 0$

$\boxed{x \geq -1} \ \wedge \ \boxed{x \neq 4} \Rightarrow \boxed{D(f_1) = \langle -1; 4) \cup (4; \infty)}$

b) $f_2: y = \sqrt{\dfrac{-3}{x^2-5x+4}}$; $D(f)$: $\dfrac{-3}{x^2-5x+4} \geq 0 \ \wedge \ x^2-5x+4 \neq 0$

$D = 25 - 4 \cdot 4 = 9$

$x_{1,2} = \dfrac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$(x-4)(x-1) \neq 0$

$x \neq 4 \wedge x \neq 1$

$ax^2 + bx + c = a \cdot (x-x_1) \cdot (x-x_2)$
$a, b, c \in R \rightarrow$ čísla
x_1, x_2 - kořeny rovnice

$-3 < 0$ vždy

tedy musí platit:

$x^2 - 5x + 4 < 0$

$(x-4)(x-1) < 0$

$(x-4<0 \wedge x-1>0) \vee (x-4>0 \wedge x-1<0)$

$(x<4 \wedge x>1) \vee (x>4 \wedge x<1)$ $\Rightarrow \underline{D(f) = (1; 4)}$

c) $f_3: y = \dfrac{-3}{\sqrt{x^2-3x}}$; $D(f)$: $x^2 - 3x \geq 0 \ \wedge \ \sqrt{x^2-3x} \neq 0 \ /^2$

$x \cdot (x-3) \geq 0 \ \wedge \ x^2 - 3x \neq 0 \Rightarrow x \cdot (x-3) > 0$

$(x>0 \wedge x-3>0) \vee (x<0 \wedge x-3<0)$

$(x>0 \wedge x>3) \vee (x<0 \wedge x<3)$

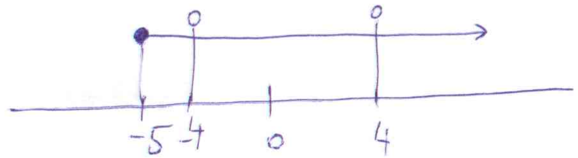
$\underline{D(f) = (3; \infty) \cup (-\infty; 0)}$

1b

1.3.5 d) $f_4: y = \dfrac{\sqrt{2x+10}}{16-x^2}$; $D(f)$: $2x+10 \geq 0 \;\wedge\; 16-x^2 \neq 0$

$$2x \geq -10 \;\wedge\; (4+x)(4-x) \neq 0$$

$$\boxed{x \geq -5} \;\wedge\; \boxed{x \neq \pm 4}$$



$\Rightarrow D(f) = \langle -5; -4) \cup (-4; 4) \cup (4; \infty)$

1.3.6 a) $f_1: y = 4^{\log(2x^2-5x-3)}$; $D(f)$: $2x^2-5x-3 > 0$

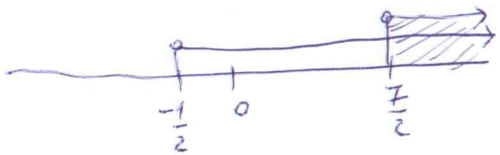
$$D = 25 + 4\cdot 2\cdot 3 = 49$$

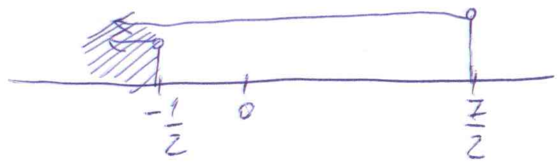
$$x_{1,2} = \frac{5 \pm 7}{4} = \begin{cases} \frac{14}{4} = \frac{7}{2} \\ -\frac{2}{4} = -\frac{1}{2} \end{cases}$$

$$2\cdot\left(x-\frac{7}{2}\right)\cdot\left(x+\frac{1}{2}\right) > 0$$

$$\left(x-\frac{7}{2} > 0 \;\wedge\; x+\frac{1}{2} > 0\right) \vee \left(x-\frac{7}{2} < 0 \;\wedge\; x+\frac{1}{2} < 0\right)$$

$$\left(x > \frac{7}{2} \;\wedge\; x > -\frac{1}{2}\right) \vee \left(x < \frac{7}{2} \;\wedge\; x < -\frac{1}{2}\right)$$





$\Rightarrow D(f) = \left(\frac{7}{2}; \infty\right) \cup \left(-\infty; -\frac{1}{2}\right)$

1.3.5 e) $f_5: y=\frac{1}{\sqrt{x}}+\sqrt{x^2-5}$ → $x^2-5\ge 0$ podľa vzorca $\boxed{(a+b)(a-b)=a^2-b^2}$ môžeme napísať:

$$(x+\sqrt{5})(x-\sqrt{5})=x^2-5$$

$$x\ge 0 \wedge \sqrt{x}\ne 0$$

$$\boxed{x>0}$$

$$\Rightarrow x^2-5\ge 0 \xLeftrightarrow[\text{vtedy}]{\text{práve}} (x+\sqrt{5})(x-\sqrt{5})\ge 0$$

$$(x+\sqrt{5}\ge 0 \wedge x-\sqrt{5}\ge 0)\vee(x+\sqrt{5}\le 0 \wedge x-\sqrt{5}\le 0)$$

$$(x\ge -\sqrt{5} \wedge x\ge \sqrt{5})\vee(x\le -\sqrt{5} \wedge x\le \sqrt{5})$$

$(\sqrt{5};\infty)$ $(-\infty;-\sqrt{5})$

ale zároveň $\boxed{x>0}$ a teda $D(f)=(\sqrt{5};\infty)$

f) $f_6: y=\frac{\sqrt{15+2x-x^2}}{8-2x}$ ⇝ $-x^2+2x+15\ge 0$

$15+2x-x^2\ge 0 \wedge 8-2x\ne 0$

$D=b^2-4ac$ $8\ne 2x$

$D=4-4\cdot 15\cdot(-1)$ $2x\ne 8$ /:2

$D=64$ $\boxed{x\ne 4}$

$$x_{1,2}=\frac{-2\pm\sqrt{64}}{2\cdot(-1)}$$

$$x_{1,2}=\frac{-2\pm 8}{-2}=\begin{cases}5\\-3\end{cases}$$

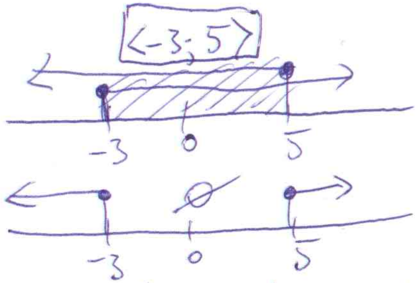
→ $ax^2+bx+c=a(x-x_1)(x-x_2)$

$-x^2+2x+15=-(x-5)(x+3)\ge 0$ /:(-1) pri násobení a delení záporným číslom sa otáča nerovnosť

$(x-5)(x+3)\le 0$

$(x-5\le 0 \wedge x+3\ge 0)\vee(x-5\ge 0 \wedge x+3\le 0)$

$(x\le 5 \wedge x\ge -3)\vee(x\ge 5 \wedge x\le -3)$



a teda $D(f)=\langle -3;4)\cup(4;5\rangle$

1.3.6 b) $f_2 : y = \ln \sqrt{\frac{3x-1}{x+4}}$

Platí:

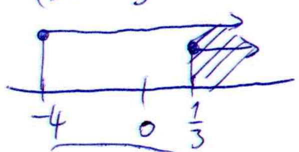
$$\boxed{\begin{array}{l} 1.\ \sqrt{f(x)} \Rightarrow f(x) \geq 0 \\ 2.\ \frac{1}{f(x)} \Rightarrow f(x) \neq 0 \\ 3.\ \log_a f(x) \Rightarrow f(x) > 0 \end{array}}$$

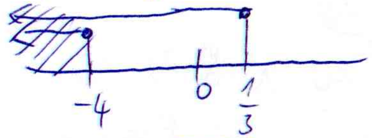
$$\boxed{\ln x = \log_e x}$$

1. $\sqrt{\frac{3x-1}{x+4}} > 0 \rightsquigarrow$ pre každú odmocninu platí, že je ≥ 0
Musíme teda vylúčiť iba prípad $\sqrt{\frac{3x-1}{x+4}} \neq 0 \iff \frac{3x-1}{x+4} \neq 0 \iff 3x-1 \neq 0 \iff 3x \neq 1 \iff \boxed{x \neq \frac{1}{3}}$

2. $x+4 \neq 0$
$\boxed{x \neq -4}$

3. $\frac{3x-1}{x+4} \geq 0 \iff (3x-1 \geq 0 \wedge x+4 \geq 0) \vee (3x-1 \leq 0 \wedge x+4 \leq 0)$
$(x \geq \frac{1}{3} \wedge x \geq -4) \vee (x \leq \frac{1}{3} \wedge x \leq -4)$





$\boxed{\langle \frac{1}{3}; \infty)}$ $\boxed{(-\infty; -4\rangle}$

všetko dokopy:
$x \neq \frac{1}{3}$
$x \neq -4$
$x \in (-\infty; -4\rangle \cup \langle \frac{1}{3}; \infty)$

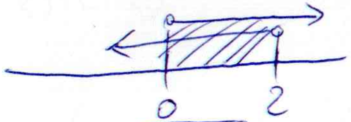
$\Rightarrow D(f) = (-\infty; -4) \cup (\frac{1}{3}; \infty)$

1.3.6 c) $f_3: y = \dfrac{-1}{\ln(2x-x^2)} \rightsquigarrow 2x-x^2>0 \wedge \ln(2x-x^2)\neq 0 \rightsquigarrow \boxed{\ln X = 0 \Leftrightarrow X=1}$!

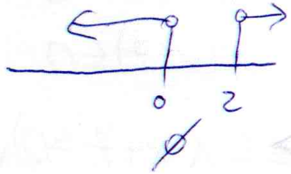
$x(2-x)>0 \wedge 2x-x^2 \neq 1$

$(x>0 \wedge 2-x>0) \vee (x<0 \wedge 2-x<0) \wedge (-x^2+2x-1\neq 0) \,/\cdot(-1)$

$(x>0 \wedge x<2) \vee (x<0 \wedge x>2) \wedge (x^2-2x+1\neq 0)$



$\boxed{(0;2)}$



$\emptyset$

$D = 4-4=0$

$x_{1,2} = \dfrac{2\pm 0}{2} = 1$

$x^2-2x+1 = (x-1)(x-1) = (x-1)^2 \neq 0$

$\boxed{x\neq 1}$

$D(f) = (0;1)\cup(1;2)$

d) $f_4: y = \log x^2 + \log(4-x^2)$

$x^2>0 \wedge 4-x^2>0$

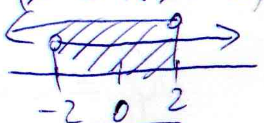
platí vždy okrem prípadu $\boxed{x\neq 0}$

(každé číslo umocnené na 2. je kladné alebo 0)

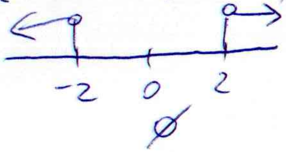
$(2-x)(2+x)>0$

$(2-x>0 \wedge 2+x>0) \vee (2-x<0 \wedge 2+x<0)$

$(x<2 \wedge x>-2) \vee (x>2 \wedge x<-2)$



$\boxed{(-2;2)}$



$\emptyset$

$D(f) = (-2;0)\cup(0;2)$

1.3.6. e) $f_5: y=\sqrt{1-\log(x^2+7x+10)} \rightsquigarrow 1-\log(x^2+7x+10)\geq 0 \quad \wedge \quad x^2+7x+10>0$

$\log(x^2+7x+10)\leq 1 \quad \wedge \quad D=49-4\cdot 10=9$

$x_{1,2}=\frac{-7\pm 3}{2}= \begin{cases}-5\\-2\end{cases}$

! $\boxed{\log_a x = y \Leftrightarrow a^y = x}$!

$\boxed{\log x = \log_{10} x}$

$x^2+7x+10\leq 10^1 \quad /-10$

$x^2+7x\leq 0$

$x(x+7)\leq 0$

$(x+5)(x+2)>0$

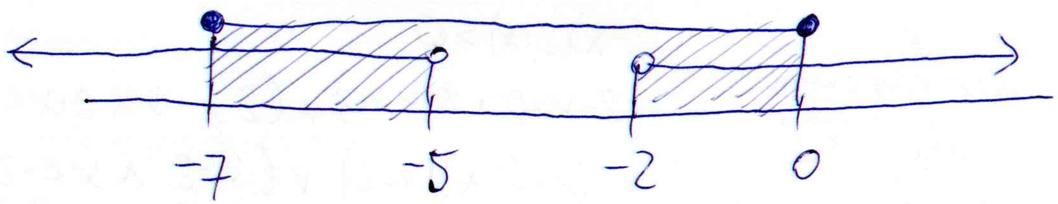
$(x\geq 0 \wedge x+7\leq 0) \vee (x\leq 0 \wedge x+7\geq 0)$

$(x\geq 0 \wedge x\leq -7) \vee (x\leq 0 \wedge x\geq -7)$

$(x+5>0 \wedge x+2>0) \vee (x+5<0 \wedge x+2<0$

$(x>-5 \wedge x>-2) \vee (x<-5 \wedge x<-2$

$\emptyset$ $\quad \boxed{\langle -7;0\rangle}$ $\quad \boxed{(-2;\infty)}$ $\quad \boxed{(-\infty;-5)}$



$D(f)=\langle -7;-5)\cup(-2;0\rangle$

1.3.6 f) $f_6: y = \dfrac{\sqrt{\ln(x-1)}}{x-2} \rightsquigarrow \ln(x-1) \geq 0 \wedge x-1>0 \wedge x-2 \neq 0$

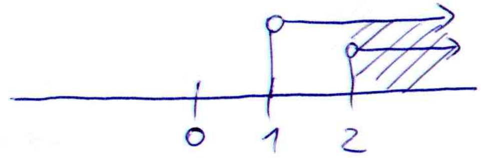
$\log_e(x-1) \geq 0 \quad \wedge \quad x>1 \quad \wedge \quad \boxed{x \neq 2}$

$\boxed{(1;\infty)}$

$x-1 \geq e^0$

! $\boxed{\overset{\circ}{x} = 1 \text{ pre všetky } x}$! $\Rightarrow$ $x-1 \geq 1$

$\boxed{x \geq 2}$ $\quad \boxed{(2;\infty)}$



$D(f) = (2;\infty)$

1.3.7 a) $f_1: y = \arcsin \dfrac{2x+4}{x}$

$\boxed{x \neq 0}$

$\cancel{-1 \leq \frac{2x+4}{x} \leq 1 \,/\cdot x}$

$\cancel{-x \leq 2x+4 \leq x}$

Ďalšie podmienky pre Definičný obor (arcusy):

4. $\arcsin X \Rightarrow X \in \langle -1;1 \rangle$

5. $\arccos X \Rightarrow X \in \langle -1;1 \rangle$

$\dfrac{2x+4}{x} \leq 1 \,/-1 \quad \wedge \quad \dfrac{2x+4}{x} \geq -1 \,/+1$

$\dfrac{2x+4}{x} - 1 \leq 0 \quad \wedge \quad \dfrac{2x+4}{x} + 1 \geq 0$

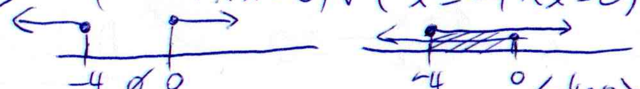
$\dfrac{2x+4}{x} - \dfrac{x}{x} \leq 0 \quad \wedge \quad \dfrac{2x+4}{x} + \dfrac{x}{x} \geq 0$

$\dfrac{2x+4-x}{x} \leq 0 \quad \wedge \quad \dfrac{2x+4+x}{x} \geq 0$

$\dfrac{x+4}{x} \leq 0 \quad \wedge \quad \dfrac{3x+4}{x} \geq 0$

$(x+4 \leq 0 \wedge x \geq 0) \vee (x+4 \geq 0 \wedge x \leq 0)$

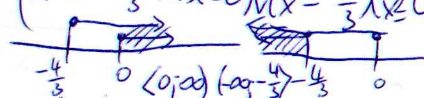
$(x \leq -4 \wedge x \geq 0) \vee (x \geq -4 \wedge x \leq 0)$



$\emptyset$ $\quad \langle -4;0)$

$(3x+4 \geq 0 \wedge x \geq 0) \vee (3x+4 \leq 0 \wedge x \leq 0)$

$(x \geq -\frac{4}{3} \wedge x \geq 0) \vee (x \leq -\frac{4}{3} \wedge x \leq 0)$

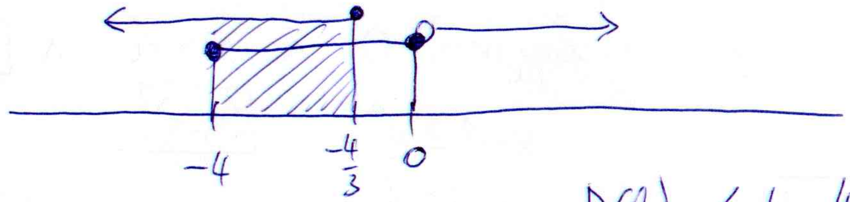


$(0;\infty)$ $\quad (-\infty;-\frac{4}{3}\rangle$

dohromady: $x \neq 0$

$\langle -4; 0 \rangle$

$\langle 0; \infty) \cup (-\infty; -\frac{4}{3}\rangle$



$D(f) = \langle -4; -\frac{4}{3} \rangle$

1.3.7 b) $f_2: y = \operatorname{arccotg} \frac{x^2}{x^2-2}$ → chyták! pre arctg a arccotg neplatia žiadne obmedzenia pre def. obor!

$x^2 - 2 \neq 0$

$x^2 \neq 2 \Rightarrow x \neq \pm\sqrt{2} \Rightarrow D(f) = (-\infty; -\sqrt{2}) \cup (-\sqrt{2}; \sqrt{2}) \cup (\sqrt{2}; \infty)$

c) $f_3: y = \arccos\left(\frac{1}{x^2}\right)$

$x^2 \neq 0 \Leftrightarrow \boxed{x \neq 0}$

$\frac{1}{x^2} \leq 1 \quad \wedge \quad \frac{1}{x^2} \geq -1$

$\frac{1}{x^2} - 1 \leq 0 \quad \wedge \quad \frac{1}{x^2} + 1 \geq 0$

$\frac{1}{x^2} - \frac{x^2}{x^2} \leq 0 \quad \wedge \quad \frac{1}{x^2} + \frac{x^2}{x^2} \geq 0$

$\frac{1-x^2}{x^2} \leq 0 \quad \wedge \quad \frac{1+x^2}{x^2} \geq 0$

→ z prvej podmienky vieme, že $x \neq 0$

a tiež platí $x^2 \geq 0$ vždy (druhá mocnina nikdy nie je záporná)

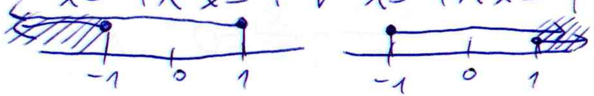
$\frac{1+x^2}{x^2} \geq 0$ teda platí vždy, lebo $\frac{1+\text{kladné č.}}{\text{kladné č.}}$ je vždy kladné

Ďalej, $\frac{1-x^2}{x^2} \leq 0 \Leftrightarrow 1 - x^2 \leq 0$ (lebo v menovateli je $x^2 \geq 0$)

podľa vzorca $a^2 - b^2 = (a+b)(a-b)$ platí: $1 - x^2 \leq 0 \Leftrightarrow (1+x)(1-x) \leq 0$

$(1+x \leq 0 \wedge 1-x \geq 0) \vee (1+x \geq 0 \wedge 1-x \leq 0)$

$x \leq -1 \wedge x \leq 1 \quad \vee \quad x \geq -1 \wedge x \geq 1$



$D(f) = (-\infty; -1\rangle \cup \langle 1; \infty)$ ⟸

1.3.7 d) $f_4: y = \frac{1}{x} + \arccos(x^2-1)$

$$\boxed{x \neq 0}$$

$x^2-1 \leq 1 \quad \wedge \quad x^2-1 \geq -1 \quad /+1$

$x^2 \leq 2 \qquad\qquad x^2 \geq 0 \Rightarrow$ platí vždy

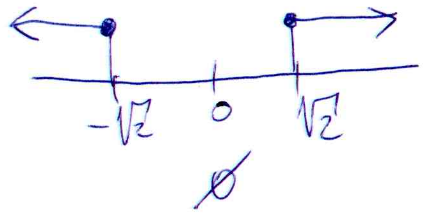
$x^2-2 \leq 0$

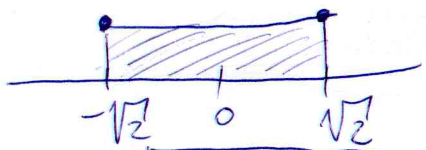
$(x+\sqrt{2})(x-\sqrt{2}) \leq 0$

$(x+\sqrt{2} \leq 0 \wedge x-\sqrt{2} \geq 0) \vee (x+\sqrt{2} \geq 0 \wedge x-\sqrt{2} \leq 0)$

$(x \leq -\sqrt{2} \wedge x \geq \sqrt{2}) \vee (x \geq -\sqrt{2} \wedge x \leq \sqrt{2})$

$\Rightarrow D(f) = \langle -\sqrt{2}; 0) \cup (0; \sqrt{2} \rangle$





$$\boxed{\langle -\sqrt{2}; \sqrt{2} \rangle}$$

e) $f_5: y = \frac{x}{\operatorname{arctg}(12-4x)}$

$\operatorname{arctg}(12-4x) \neq 0$

$12-4x \neq 0$

platí: $\operatorname{arctg} X = 0 \Leftrightarrow X = 0$ (dá sa nájsť v tabuľkách)

$$\boxed{x \neq 3}$$

$D(f) = (-\infty; +3) \cup (+3; \infty)$

(1.3.7) f) $f_6: y=\sqrt{\arcsin(x-4)}$

$-1 \le x-4 \le 1 \quad /+4$ $\quad\wedge\quad$ $\arcsin(x-4) \ge 0$

$3 \le x \le 5$

$\boxed{\langle 3;5\rangle}$

môžeme počítať takto naraz, ak tam máme iba $\underline{x}$ a nie $\underline{x^2}$ a ak $\underline{x}$ nie je v menovateli

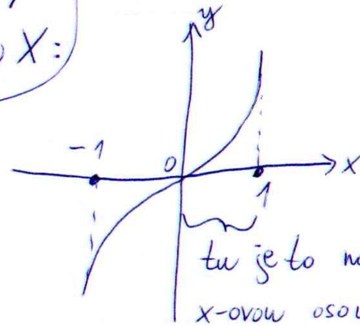
$0 \le x-4 \le 1 \quad /+4$

$4 \le x \le 5$

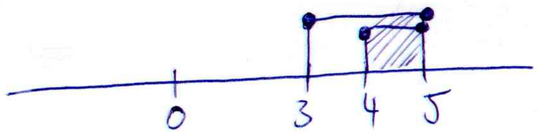
$\boxed{\langle 4;5\rangle}$

$\arcsin X \ge 0 \Leftrightarrow X \in \langle 0;1\rangle$

toto vidieť z grafu $\arcsin X$:



tu je to nad x-ovou osou a teda ≥ 0



$\underline{\underline{D(f)=\langle 4;5\rangle}}$

(1.3.8) a) $f_1: y=\log(1-2x)-3\arcsin\frac{3x-1}{2}$

$1-2x>0$ $\quad\wedge\quad$ $-1 \le \frac{3x-1}{2} \le 1 \quad /\cdot 2$

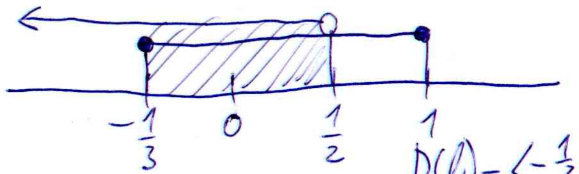
$\boxed{x<\frac{1}{2}}$

$-2 \le 3x-1 \le 2 \quad /+1$

$-1 \le 3x \le 3 \quad /:3$

$-\frac{1}{3} \le x \le 1$

$\boxed{\langle -\frac{1}{3};1\rangle}$



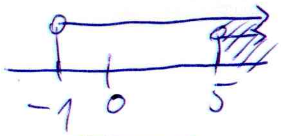
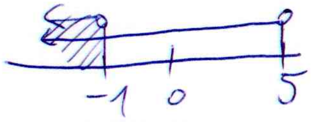
$\underline{\underline{D(f)=\langle -\frac{1}{3};\frac{1}{2})}}$

1.3.8 b) $f_2: y = 5\cdot\log\left(\frac{x+1}{x-5}\right) - \frac{\sqrt{5x-10}}{x^2-36}$

$\frac{x+1}{x-5} > 0 \quad \wedge \quad x-5 \neq 0 \quad \wedge \quad 5x-10 \geq 0 \quad \wedge \quad x^2-36 \neq 0$

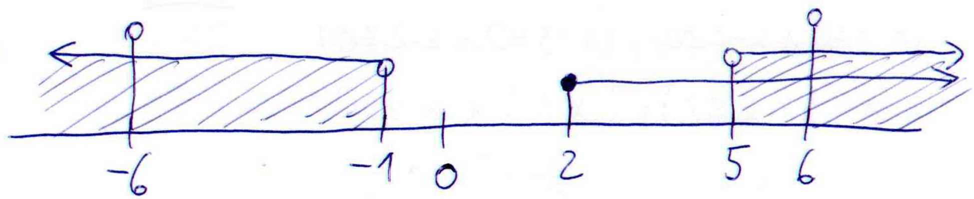
$(x+1>0 \wedge x-5>0) \vee (x+1<0 \wedge x-5<0) \qquad \boxed{x \neq 5} \qquad 5x \geq 10 \quad \wedge \quad (x-6)(x+6) \neq 0$

$(x>-1 \wedge x>5) \vee (x<-1 \wedge x<5) \qquad \boxed{x \geq 2} \qquad \boxed{x \neq \pm 6}$

$\boxed{(5;\infty)} \cup \boxed{(-\infty;-1)}$

všetko dokopy:



$D(f) = (-\infty;-6) \cup (-6;-1) \cup (5;6) \cup (6;\infty)$

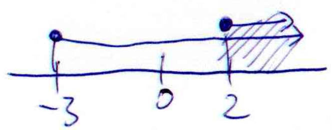
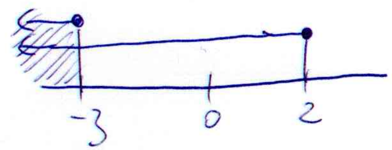
c) $f_3: y = \arccos(3+2x) + \sqrt{\frac{x-2}{x+3}}$

$-1 \leq 3+2x \leq 1 \quad /-3 \qquad \wedge \qquad x+3 \neq 0 \qquad \wedge \qquad \frac{x-2}{x+3} \geq 0$

$-4 \leq 2x \leq -2 \quad /:2 \qquad \boxed{x \neq -3} \qquad (x-2 \geq 0 \wedge x+3 \geq 0) \vee (x-2 \leq 0 \wedge x+3 \leq 0)$

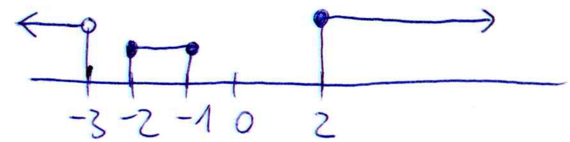
$-2 \leq x \leq -1 \qquad (x \geq 2 \wedge x \geq -3) \vee (x \leq 2 \wedge x \leq -3)$

$\boxed{\langle -2;-1 \rangle}$

$\boxed{\langle 2;\infty) \cup (-\infty;-3\rangle}$

Dohromady:



$D(f) = \emptyset$

1.3.8 d) $f_4: y = \dfrac{\sqrt{x^2-5x+6}}{\ln(2x-5)} - \sqrt{5-x}$

$x^2-5x+6 \geq 0 \quad \wedge \quad \ln(2x-5) \neq 0 \quad \wedge \quad 2x-5>0 \quad \wedge \quad 5-x \geq 0$

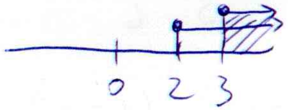
$D = 25-24 = 1$

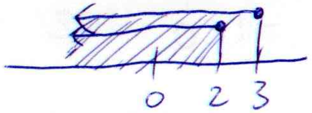
$x_{1,2} = \dfrac{5 \pm 1}{2} = \left\langle \begin{matrix} 3 \\ 2 \end{matrix} \right.$

$(x-3)(x-2) \geq 0$

$(x-3 \geq 0 \wedge x-2 \geq 0) \vee (x-3 \leq 0 \wedge x-2 \leq 0)$

$(x \geq 3 \wedge x \geq 2) \vee (x \leq 3 \wedge x \leq 2)$





$\boxed{\langle 3; \infty) \quad \cup \quad (-\infty; 2\rangle}$

$\log_e(2x-5) \neq 0$

$2x-5 \neq e^0$

$2x-5 \neq 1$

$2x \neq 6$

$\boxed{x \neq 3}$

$2x > 5$

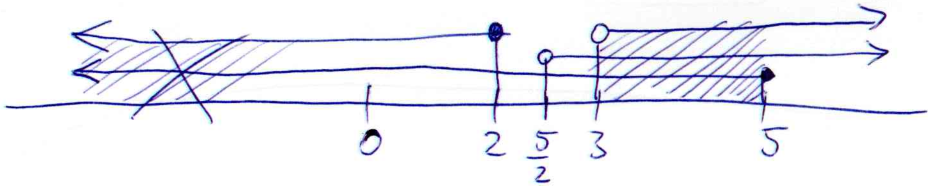
$x > \frac{5}{2}$

$\boxed{\left(\frac{5}{2}; \infty\right)}$

$x \leq 5$

$\boxed{(-\infty; 5\rangle}$

Dohromady:



$D(f) = (3; 5\rangle$

1.3.8

e) $f_5: y = \dfrac{\sqrt{x^2-x-2}}{\ln x} - 4\cdot \arcsin\dfrac{1-2x}{4}$

$x^2-x-2 \geq 0 \ \wedge\ \ln x \neq 0 \ \wedge\ x>0 \ \wedge\ -1 \leq \dfrac{1-2x}{4} \leq 1 \ /\cdot 4$

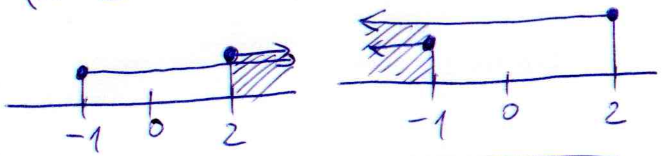
$D = 1+4\cdot 2 = 9$

$x_{1,2} = \dfrac{1\pm 3}{2} = \left\langle \begin{array}{l} 2 \\ -1 \end{array}\right.$

$(x-2)(x+1) \geq 0$

$(x-2\geq 0 \wedge x+1 \geq 0) \vee (x-2 \leq 0 \wedge x+1 \leq 0)$

$(x \geq 2 \wedge x \geq -1) \vee (x \leq 2 \wedge x \leq -1)$



$\boxed{\langle 2;\infty) \quad \cup \quad (-\infty;-1\rangle}$

$\log_e x \neq 0$

$x \neq e^0$

$\boxed{x \neq 1}$

$\boxed{(0;\infty)}$

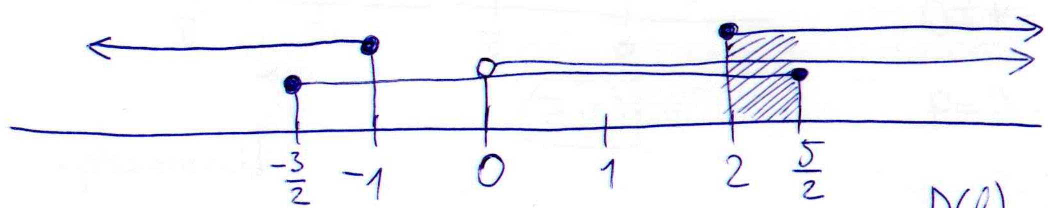
$-4 \leq 1-2x \leq 4 \ /-1$

$-5 \leq -2x \leq 3 \ /:(-2)$

$\dfrac{5}{2} \geq x \geq -\dfrac{3}{2}$

$\boxed{\left\langle -\dfrac{3}{2}; \dfrac{5}{2}\right\rangle}$

Dohromady:



$D(f) = \left\langle 2; \dfrac{5}{2}\right\rangle$

1.3.8

f) $f_6: y=\sqrt{\log\frac{5x-x^2}{4}}+\frac{1}{\log_2 x-2}$

$\log\frac{5x-x^2}{4}\geq 0 \;\wedge\; \frac{5x-x^2}{4}>0 \;\wedge\; \log_2 x-2\neq 0 \;\wedge\; x>0$

$\frac{5x-x^2}{4}\geq 10^0$

$\frac{5x-x^2}{4}\geq 1 \;/\cdot 4$

$5x-x^2\geq 4$

$-x^2+5x-4\geq 0 \;/\cdot(-1)$

$x^2-5x+4\leq 0$

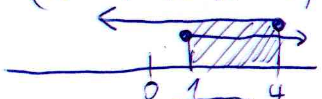
$D=25-16=9$

$x_{1,2}=\frac{5\pm 3}{2}=\begin{cases}4\\1\end{cases}$

$(x-4)(x-1)\leq 0$

$(x-4\leq 0 \wedge x-1\geq 0)\vee(x-4\geq 0 \wedge x-1\leq 0)$

$(x\leq 4 \wedge x\geq 1)\vee(x\geq 4 \wedge x\leq 1)$



$\boxed{\langle 1;4\rangle}$

$\emptyset$

$\frac{5x-x^2}{4}>0$

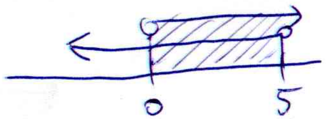
$\Updownarrow$

$5x-x^2>0$

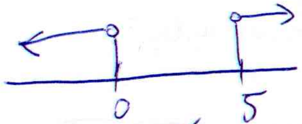
$x(5-x)>0$

$(x>0 \wedge 5-x>0)\vee(x<0 \wedge 5-x<0)$

$(x>0 \wedge x<5)\vee(x<0 \wedge x>5)$



$\boxed{(0;5)}$



$\emptyset$

$\log_2 x-2\neq 0$

$\log_2 x\neq 2$

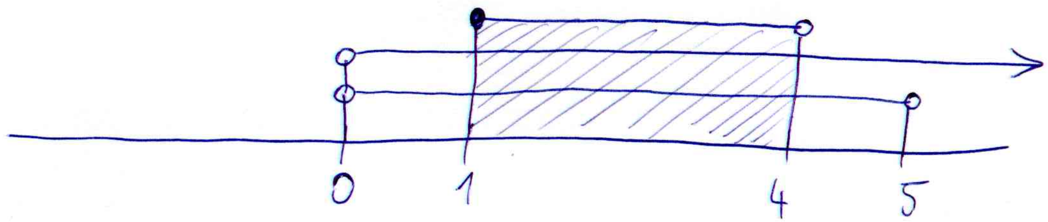
$x\neq 2^2$

$\boxed{x\neq 4}$

$x>0$

$\boxed{(0;\infty)}$

Dohromady:



$D(f)=\langle 1;4)$

g) $f_7: y = \arccos\dfrac{3}{2x-5} + \ln(6+11x-2x^2)$

$$\frac{3}{2x-5} \geq -1 \quad \wedge \quad \frac{3}{2x-5} \leq 1 \quad \wedge \quad -2x^2+11x+6>0 \quad \wedge \quad 2x-5 \neq 0$$

$\boxed{x \neq \frac{5}{2}}$

$$\frac{3}{2x-5}+1 \geq 0 \qquad \frac{3}{2x-5}-1 \leq 0$$

$$\frac{3}{2x-5}+\frac{2x-5}{2x-5} \geq 0 \qquad \frac{3}{2x-5}-\frac{2x-5}{2x-5} \leq 0$$

$$\frac{3+2x-5}{2x-5} \geq 0 \qquad \frac{3-2x+5}{2x-5} \leq 0$$

$$\frac{2x-2}{2x-5} \geq 0 \,/:2 \qquad \frac{8-2x}{2x-5} \leq 0 \,/:2$$

$$\frac{x-1}{2x-5} \geq 0 \qquad \frac{4-x}{2x-5} \leq 0$$

$D = 121 + 4\cdot 2\cdot 6 = 169$

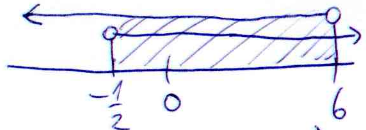
$x_{1,2} = \dfrac{-11 \pm 13}{-4} = \begin{cases} -\frac{1}{2} \\ 6 \end{cases}$

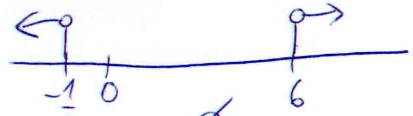
$-2(x-6)(x+\frac{1}{2}) > 0 \,/:(-2)$

$(x-6)(x+\frac{1}{2}) < 0$

$(x-6<0 \wedge x+\frac{1}{2}>0) \vee (x-6>0 \wedge x+\frac{1}{2}<0)$

$(x<6 \wedge x>-\frac{1}{2}) \vee (x>6 \wedge x<-\frac{1}{2})$



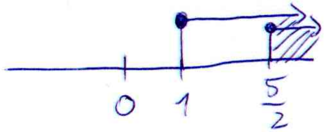


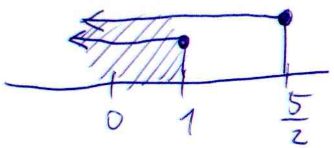
$\emptyset$

$\boxed{(-\frac{1}{2}; 6)}$

$(x-1\geq 0 \wedge 2x-5\geq 0) \vee (x-1\leq 0 \wedge 2x-5\leq 0)$

$(x\geq 1 \wedge x\geq \frac{5}{2}) \vee (x\leq 1 \wedge x\leq \frac{5}{2})$

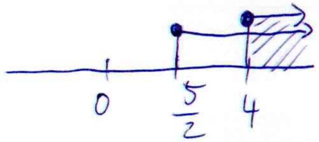


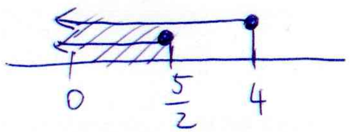


$\boxed{\langle \frac{5}{2}; \infty) \cup (-\infty; 1\rangle}$

$(4-x\leq 0 \wedge 2x-5\geq 0) \vee (4-x\geq 0 \wedge 2x-5\leq 0)$

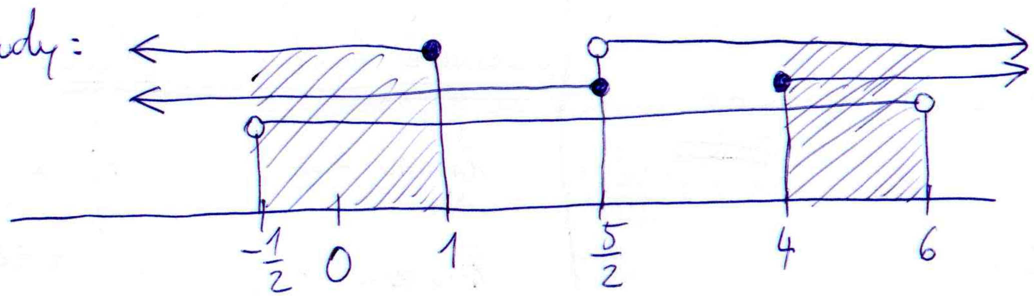
$(x\geq 4 \wedge x\geq \frac{5}{2}) \vee (x\leq 4 \wedge x\leq \frac{5}{2})$





$\boxed{\langle 4; \infty) \cup (-\infty; \frac{5}{2}\rangle}$

Dohromady:



$D(f) = (-\frac{1}{2}; 1\rangle \cup \langle 4; 6)$

1.3.8

h) $f_8: y = \dfrac{14-x}{\ln(x^2-4)} + \arccos(3x+7)$

$\ln(x^2-4) \neq 0 \quad \wedge \quad x^2-4 > 0 \quad \wedge \quad -1 \le 3x+7 \le 1 \rightsquigarrow -8 \le 3x \le -6 \quad /:3$

$-\frac{8}{3} \le x \le -2$

$\boxed{\langle -\frac{8}{3}; -2 \rangle}$

$x^2-4 \neq 1$

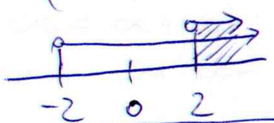
$x^2 \neq 5$

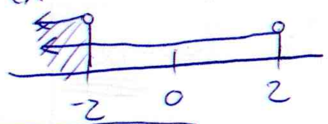
$\boxed{x \neq \pm\sqrt{5}}$

$(x-2)(x+2) > 0$

$(x-2>0 \wedge x+2>0) \vee (x-2<0 \wedge x+2<0)$

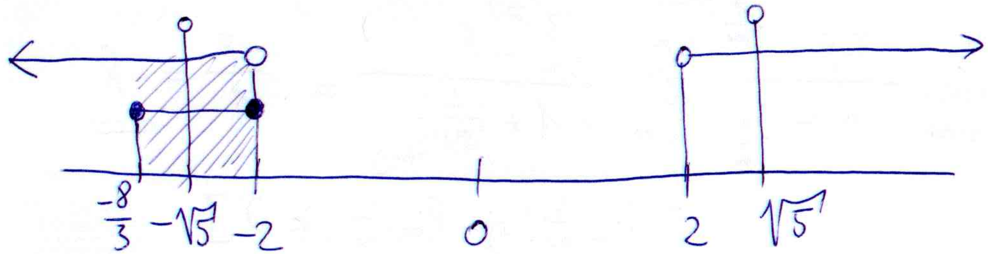
$(x>2 \wedge x>-2) \vee (x<2 \wedge x<-2)$





$\boxed{(2; \infty) \cup (-\infty; -2)}$

Dohromady:



$D(f) = \langle -\frac{8}{3}; -\sqrt{5}) \cup (-\sqrt{5}; -2)$